SPECIALIST MATHEMATICS UNIT 4
SAC 3: INTEGRAL CALCULUS AND APPLICATIONS TEST

NAME: ______________________________

PAPER TWO: Technology Active **Time: 25 Minutes** **Total = 18 marks**

SECTION A: MULTIPLE CHOICE

Please circle the correct answer.

Question 1

An antiderivative of $\frac{1}{x^2-2x+2}$ is:

A. $-\left(x^2-2x+2\right)^{-2}$

B. $\log_e(x^2-2x+2)$

C. $\log_e\left|\frac{x-2}{x+1}\right|$

D. $\operatorname{arcsec}(x-1)$

E. $\arctan(x-1)$

Question 2

A solid is constructed by rotating the function $y=1-\cos(2x)$, where $0\le x\le\frac{\pi}{2}$, about the y-axis. The volume of this solid is:

A. $\frac{\pi\left(\pi^2-4\right)}{4}$

B. $\frac{-\pi\left(\pi^2-20\right)}{4}$

C. $\frac{\pi^3}{2}$

D. $\frac{-\pi\left(\pi^2-4\pi-4\right)}{4}$

E. $\frac{\pi\left(\pi^2+4\right)}{4}$

Question 3

The region enclosed by the graph of $y = x^2 + 1$ and he lines $y = 1$ and $y = 4$ is rotated about the y-axis to form a solid of revolution. The volume of the solid is given by

A. $\pi \int_{0}^{\sqrt{3}} \left(x^2 + 1\right)\, dx$

B. $\pi \int_{1}^{4} \left(y - 1\right)\, dx$

C. $\pi \int_{1}^{4} \left(x^2 + 1\right)\, dx$

D. $\pi \int_{1}^{4} \left(y - 1\right)\, dy$

E. $\pi \int_{0}^{\sqrt{3}} \left(y - 1\right)\, dy$

Question 4

Using a suitable substitution, $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\tan(x)\log_e\left(\sec(x)\right)\right)dx$ can be expressed completely in terms of *u* as:

A. $\int_{\frac{2}{\sqrt{3}}}^{2} \left(\log_e(u)\right)du$

B. $\int_{-\log\left(\frac{\sqrt{3}}{2}\right)}^{\log_e(2)} (u)\,du$

C. $-\int_{-\frac{\sqrt{3}}{2}}^{\frac{1}{2}} (u)\,du$

D. $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\log_e(u)\right)du$

E. $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (u)\,du$

Question 5

If the substitutions $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\, dx$ becomes:

A. $\int_1^2 \frac{1-u^2}{u}\, du$

B. $\int_2^4 \frac{1-u^2}{u}\, du$

C. $\int_1^2 \frac{1-u^2}{2u}\, du$

D. $\int_1^2 \frac{1-u^2}{4u}\, du$

E. $\int_2^4 \frac{1-u^2}{2u}\, du$

SECTION B: SHORT ANSWER/ANALYSIS

Question 6 (7 marks)

A wine glass is formed by rotating, around the *y*-axis, the graph defined by function

$f:[0,2] \to R,\ f(x) = \frac{1}{10}\left(2+5x^3\right)$. All measurements are in cm.

a) Sketch the graph of $f(x)$ clearly labelling coordinates of endpoints.

2 marks

b) State a definite integral that would find the volume of the glass formed, when full, after it is rotated around the *y*-axis.

2 marks

c) Evaluate this volume, in cubic centimetres.

1 mark

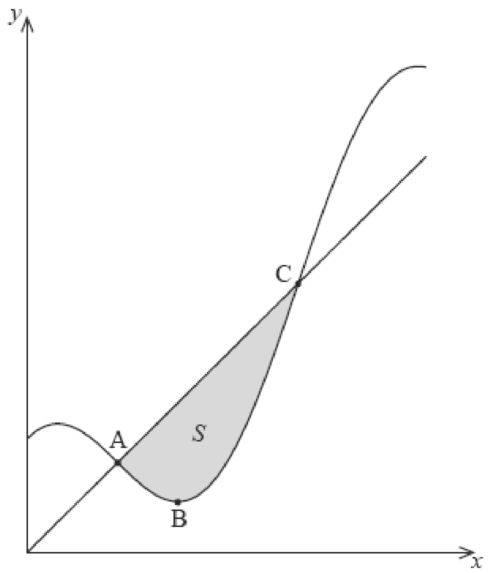
The curve $f:[0,a] \to R,\ f(x) = \frac{1}{10}\left(2+5x^3\right)$ is rotated about the *x*-axis now and the volume of the solid obtained in this way is equal by $\frac{22\pi}{175}$ cubic centimetres.

d) Find the value od a.

2 marks

Question 7 (6 marks)

Let f be a function defined by $f(x) = x + 2\cos x$, $x \in [0, 2\pi]$. The diagram below shows a region S bound by the graph of f and the line $y = x$.



A and C are the points of intersection of the line $y = x$ and the graph of f, and B is the minimum point of f.

a) If A, B and C have x-coordinates $\frac{a\pi}{2}, \frac{b\pi}{2}$ and $\frac{c\pi}{2}$, where $a, b, c \in Z^+$, find the values of a, b and c.

3 marks

b) Write down a definite integral which would find the area of region S.

2 marks

c) Hence find the area of the region S.

1 mark

END OF PAPER TWO