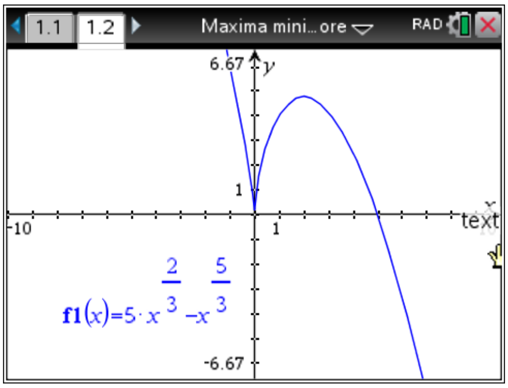
MAXIMA, MINIMA AND POINTS OF INFLECTION EXPLORATION ANSWERS

1. Sketch the following graph on GDC: $y = 5x^{\frac{2}{3}} - x^{\frac{5}{3}}$.

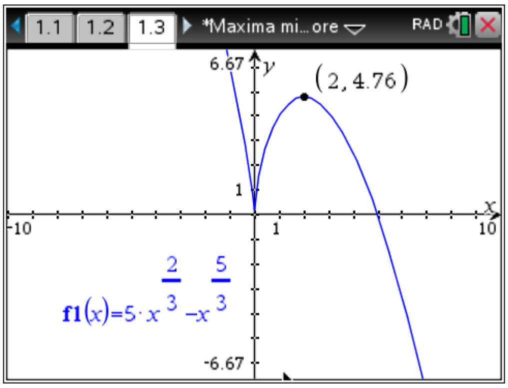


2. Find an expression for $\frac{dy}{dx}$. Take out the common factors including $x^{-\frac{1}{3}}$. Solve the equation $\frac{dy}{dx} = 0$.

$$\frac{dy}{dx} = \frac{10}{3x^{\frac{1}{3}}} - \frac{5x^{\frac{2}{3}}}{3}$$

$$= \frac{-5(x-2)}{3x^{\frac{1}{3}}}$$

$$\frac{dy}{dx} = 0 \Rightarrow x = 2$$

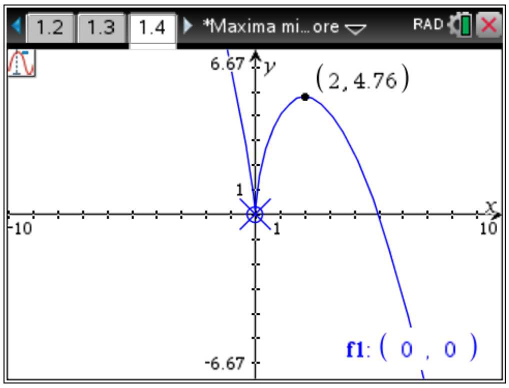


3. For what value of x is the first derivative equal to zero? What is true about the graph of the function at that value of x?

There is a stationary point at $x = 2$.

4. Look closely at the expression you obtained for the first derivative. Is there any value of x for which the first derivative is undefined? What is true about the graph at this value of x?

When $x = 0$, the first derivative is undefined. A sharp point on the graph.



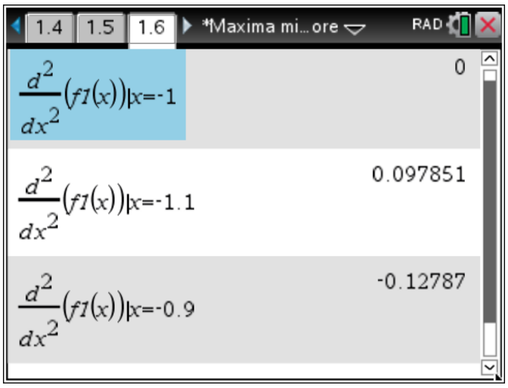
5. Find the expression for the second derivative $\frac{d^2y}{dx^2}$.

Factorise this expression.

$$\frac{d^2y}{dx^2} = -\frac{10(x+1)}{9x^{\frac{4}{3}}}$$

6. If $\frac{d^2y}{dx^2} < 0$, the graph is concave down. Show algebraically that the graph is concave down at $x = 1$. Tell what it means graphically to be concave down.

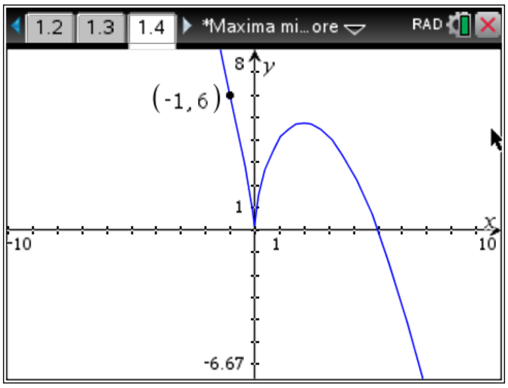
When $x = 1$, $\frac{d^2x}{dx^2} = -\frac{20}{9} < 0$.



7. Find the value of x at which $\frac{d^2y}{dx^2} = 0$. The corresponding point on the graph is called a point of inflection. Mark this point on your graph.

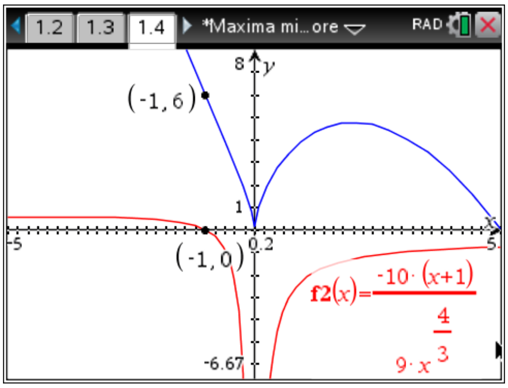
$$\frac{d^2y}{dx^2} = -\frac{10(x+1)}{9x^{\frac{4}{3}}} = 0 \Rightarrow x = -1$$

The inflection point is (-1, 6)



8. By picking a value of *x* on either side of the point of inflection in Question 7, show that the graph is really concave up on one side and concave down on the other, even though the graph appears to be straight in the neighbourhood of this *x*-value.

The graph of the second derivative:



The second derivative is positive to the left of x = -1 and negative to the right of this point. It means that concavity changes.

9. Now draw a graph of the first derivative on your calculator. What is true about the first derivative to the left and to the right of the point of inflection?

The first derivative is negative at both sides of the point of inflection and it reaches maximum at x = -1.

10. Summarise the necessary and sufficient conditions for a function to have a point of inflection. Hint: It can be done in two different ways.

The second derivative is zero and the second derivative changes sign (meaning that the function changes the direction of its concavity) at the point of inflection.

The second derivative is zero and the first derivative maintains sign on both sides of the inflection point with a local maximum or minimum at this point. In other words inflection points mark the places on the curve $y = f(x)$ where the rate of change of y with respect to x changes from increasing to decreasing or vice versa.