International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 2

1.5 hours

NAME:_____

November 2015

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL*** *information booklet* is required for this paper.

SECTION A: 54 MARKS
SECTION B: 32 MARKS
TOTAL: 86 MARKS

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. [Maximum mark: 5]

Consider the function $f(x) = e^{x-2} - 3,\ x \in \mathbb{R}$.

(a) Find the inverse function $f^{-1}(x)$. [3 marks]

(b) Find the area of the region enclosed by the graphs of $f(x)$ and $f^{-1}(x)$. [2 marks]

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2. [Maximum mark: 7]

Emma acquires a new cell phone for her birthday and receives texts from her friends. It is assumed that the daily number of texts Emma receives follows a Poisson distribution with mean $m = 5$.

(a) (i) Find the probability that on a certain day Emma receives more than 7 texts.

(ii) Determine the expected number of days in a week on which Emma receives more than 7 texts. [4]

(b) Find the probability that Emma receives fewer than 30 texts during a week. [3]

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3. [Maximum mark: 7]

A team of 6 players is to be selected from 10 volleyball players, of whom 8 are boys and 2 are girls.

(a) In how many ways can the team be selected? [2]

(b) In how many of these selections is exactly one girl in the team? [3]

(c) If the selection of the team is made at random, find the probability that exactly one girl is in the team. [2]

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4. [Maximum mark: 6]

a) Sketch the graph of $f(x) = \begin{cases} \sqrt{x-1}+2, & x>1 \\ x-1, & x \le 1 \end{cases}$ [3]

b) Is $f(x)$ a continuous function? Explain. [3]

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5. [Maximum mark: 6]

Find the value of c for which the roots of $2x^2 - 21x + c = 0$ are in the ratio 1:2.

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6. [Maximum mark: 6]

Find the term independent of x in the expansion of $\left(2x^3 - \frac{1}{4x^6}\right)^6$.

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7. [Maximum mark: 6]

The graph of $y = \ln(5x+10)$ is obtained from the graph of $y = \ln x$ by a translation of a units in the direction of the x-axis followed by a translation of b units in the direction of the y-axis.

(a) Find the value of a and the value of b. [4]

The region bounded by the graph of $y = \ln(5x+10)$, the x-axis and the lines $x = e$ and $x = 2e$ is rotated through 2π radians about the x-axis.

(b) Find the volume generated. [2]

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8. [Maximum mark: 5]

Consider the function $f(x) = xe^{-x}$.

Find the coordinates of the point of inflexion.

9. [Maximum mark: 6]

Natasha lives in Chicago and has relatives in Nashville and St. Louis.
Each time she visits her relatives, she either flies or drives.

When travelling to Nashville, the probability that she drives is $\frac{4}{5}$ and when travelling to St. Louis, the probability that she flies is $\frac{1}{3}$.

Given that the probability that she drives when visiting her relatives is $\frac{13}{18}$, find the probability that for a particular trip,

(a) she travels to Nashville; [3]

(b) she is on her way to Nashville, given that she is flying. [3]

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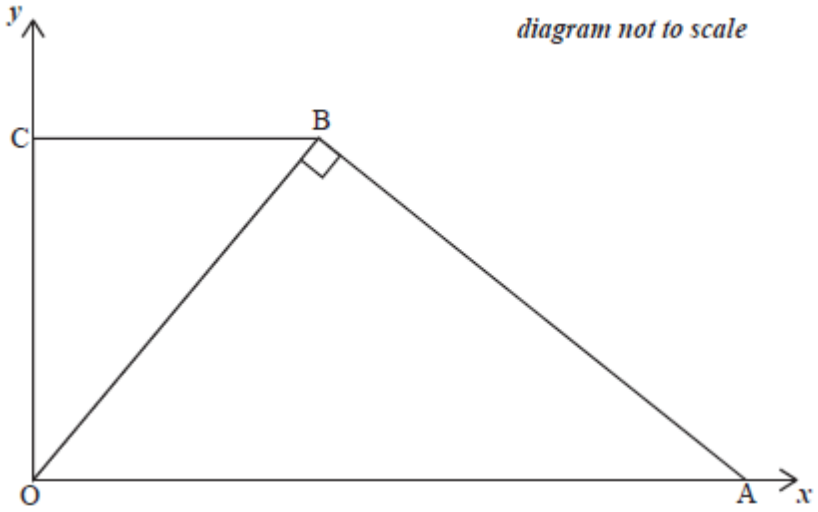
Do **not** write solutions on this page.

Section B

Answer **all** questions on lined paper provided. Please start each question on a new page.

10. [Maximum mark: 11]

The following diagram shows two triangles, OBC and OBA, on a set of axes. Point C lies on the *y*-axis, and O is the origin.



The equation of the line BC is $y = 4$.

(a) Write down the coordinates of point C. [1]

The *x*-coordinate of point B is *a*.

(b) (i) Write down the coordinates of point B;

(ii) Write down the gradient of the line OB. [2]

Point A lies on the *x*-axis and the line AB is perpendicular to line OB.

(c) (i) Write down the gradient of line AB.

(ii) Show that the equation of the line AB is $4y + ax - a^2 - 16 = 0$. [3]

The area of triangle AOB is **three times** the area of triangle OBC.

(d) Calculate the value of *a*. [5]

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11. [Maximum mark: 11]

Consider the triangle PQR where $Q\hat{P}R = 30^o$, $PQ = (x+2)$ cm and $PR = (5-x)^2$ cm.

(a) Show that the area, A cm^2, of the triangle is given by $A = \frac{1}{4}\left(x^3 - 8x^2 + 5x + 50\right)$. [2]

(b) (i) State $\frac{dA}{dx}$.

(ii) Verify that $\frac{dA}{dx} = 0$ when $x = \frac{1}{3}$. [3]

(c) (i) Find $\frac{d^2A}{dx^2}$ and hence justify that $x = \frac{1}{3}$ gives the maximum area of triangle PQR.

(ii) State the maximum area of triangle PQR.

(iii) Find QR when the area of triangle PQR is a maximum. [6]

12. [Maximum mark: 10]

A particle moves in a straight line, its velocity v ms^{-1} at time t seconds is given by $v = 9t - 3t^2, 0 \le t \le 5$.

At time $t = 0$, the displacement s of the particle from an origin O is 3 m.

(a) Find the displacement of the particle when $t = 4$. [3]

(b) Sketch a displacement-time graph for the particle, $0 \le t \le 5$, showing clearly where the curve meets the axes and the coordinates of the points where the displacement takes greatest and least values. [5]

(c) Find the distance travelled in the first 5 seconds of the motion. [2]

END OF PAPER 2