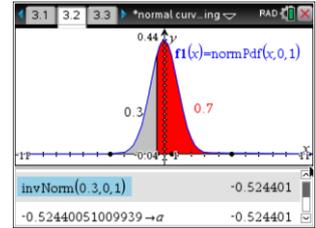


IB HL MATHEMATICS CONTINUOUS PROBABILITY ASSIGNMENT



Question 1

Consider the function

$$f(x) = \begin{cases} \frac{27}{4}(2x - 3x^2), & 0 \leq x \leq \frac{2}{3} \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Sketch the graph of $f(x)$.
- (b) Show that $f(x)$ is a probability density function.

Question 2

X is a random variable with a probability density function given by

$$f(x) = \begin{cases} a(3x + 2)^3, & -\frac{2}{3} \leq x \leq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the value of a .
- (b) Find the exact value of $E(X)$.
- (c) Find the probability of X being greater than the mean.

Question 3

X is a random variable with a probability density function given by

$$f(x) = \begin{cases} \frac{4}{25\pi} \sqrt{25 - x^2}, & 0 \leq x \leq 5, \\ 0, & \text{elsewhere.} \end{cases}$$

Use your calculator to find correct to 4 decimal places:

- (a) the mean of X
- (b) the variance of X
- (c) the standard deviation of X
- (d) the probability that X is within two standard deviations from the mean.

Question 4

An agent states that his young cattle have weights, which are normally distributed with a mean of 300 kg and a standard deviation of 40 kg. A farmer wishes to purchase 100 cattle from this agent, but is interested in cattle whose weight is between 250 and 320 kg. What percentage of the 100 cattle is within this range?

Question 5

In a factory producing glasses, the weights of glasses are known to have a mean of 160 grams. It is also known that the interquartile range of the weights of glasses is 28 grams. Assuming the weights of glasses to be normally distributed, find the standard deviation of the weights of glasses.

Question 6

The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ . 15% of the cars travelled at speeds greater than

90 km h⁻¹ and 12% of them at speeds less than 40 km h⁻¹. Find μ and σ .

Question 7

Consider the probability density function

$$f(x) = \begin{cases} kx^2, & \text{for } 0 \leq x \leq 0.5 \\ 0.5k(1-x), & \text{for } 0.5 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

- Find the value of k .
- Sketch the graph of the distribution.
- Find $P(X < 0.7)$.