# MAXIMA, MINIMA AND POINTS OF INFLECTION EXPLORATION

**Objective:** Find the first and second derivatives for a function and show how values of these correspond to the graph.

1. Sketch the following graph on your CAS calculator: $y = 5x^{\frac{2}{3}} - x^{\frac{5}{3}}$.

2. Find an expression for $\frac{dy}{dx}$. Take out the common factors including $x^{-\frac{1}{3}}$. Solve the equation $\frac{dy}{dx} = 0$.

3. For what value of *x* is the first derivative equal to zero? What is true about the graph of the function at that value of *x*?

4. Look closely at the expression you obtained for the first derivative. Is there any value of *x* for which the first derivative is undefined? What is true about the graph at this value of *x*?

5. Find the expression for the second derivative $\frac{d^2y}{dx^2}$. Factorise this expression.

6. If $\frac{d^2y}{dx^2} < 0$, the graph is concave down. Show algebraically that the graph is concave down at $x = 1$. Tell what it means graphically to be concave down.

7. Find the value of *x* at which $\frac{d^2y}{dx^2} = 0$. The corresponding point on the graph is called a point of inflection. Mark this point on your graph.

8. By picking a value of *x* on either side of the point of inflection in Question 7, show that the graph is really concave up on one side and concave down on the other, even though the graph appears to be straight in the neighbourhood of this *x*-value.

9. Now draw a graph of the first derivative on your calculator. What is true about the first derivative to the left and to the right of the point of inflection?

10. Summarise the necessary and sufficient conditions for a function to have a point of inflection. Hint: It can be done in two different ways.