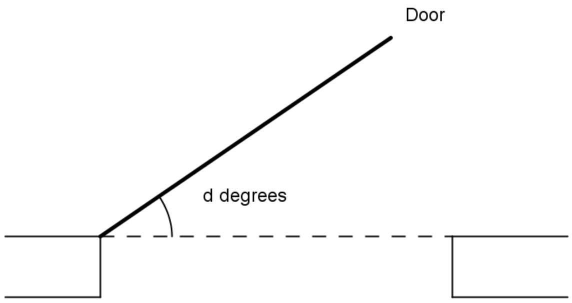
Instantaneous rate of change.

Motion of a door with an automatic closer.

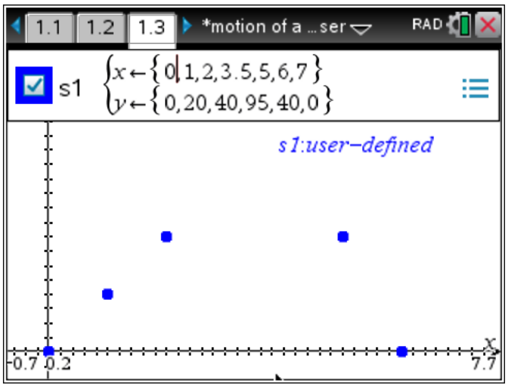


The diagram shows a door with an automatic closer. At time t=0 seconds someone pushes the door. It swings open, slows down, stops, starts closing, then slams shut at time t=7 seconds.

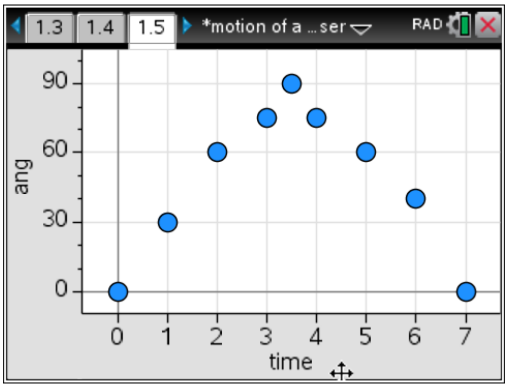
As the door is in motion, the number of degrees, d, it is from its closed position varies with time t.

1. Sketch a reasonable graph of d versus d.

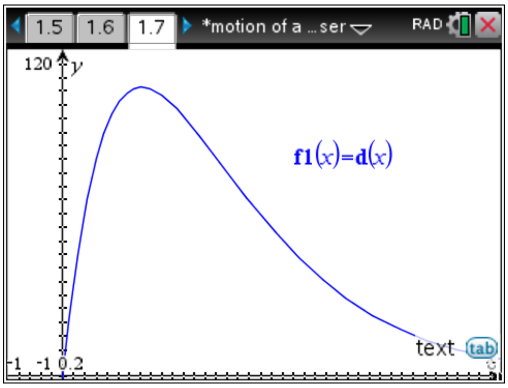
Scatterplot:



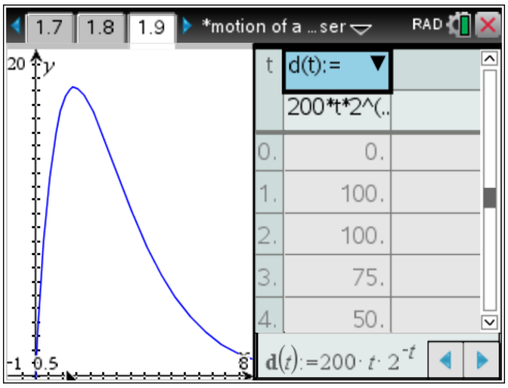
Or spreadsheet and data graph:



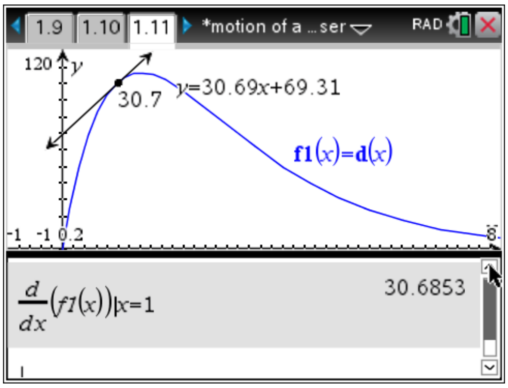
2. Suppose that d is given by the equation $d(t) = 200t \times 2^{-t}$. Plot this graph on your calculator and copy to your book.



3. Make a table of values of d for each second from t=0 through t=10. Round to the neares 0.1 of a degree.

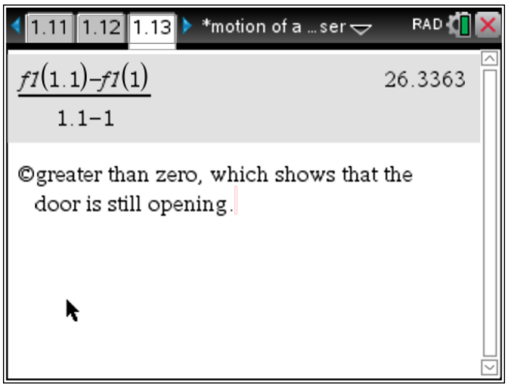


4. At time t=1 second, does the door appear to be opening or closing? How do you tell?

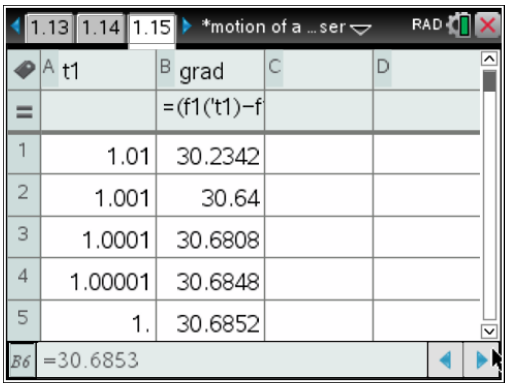


5. What is the average rate at which the door is moving for the time interval [1,1.1]?

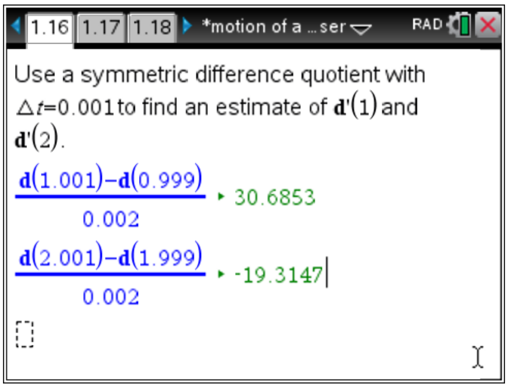
Based on your answer, does the door seem to be opening or closing at time t=1? Explain.



6. Find an estimate of the instantaneous rate at which the door is moving at time t=1, taking a numerical approach.

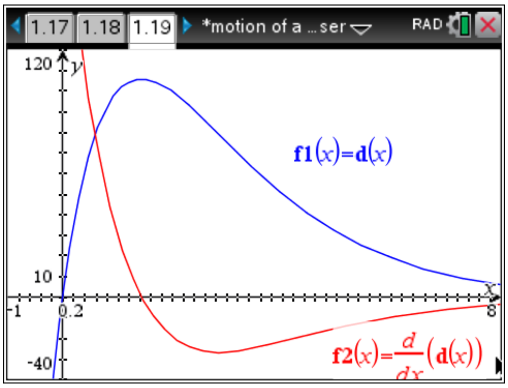


7. Use a symmetric difference quotient with $\Delta t=0.001$to find an estimate of d'(1) and d'(2).

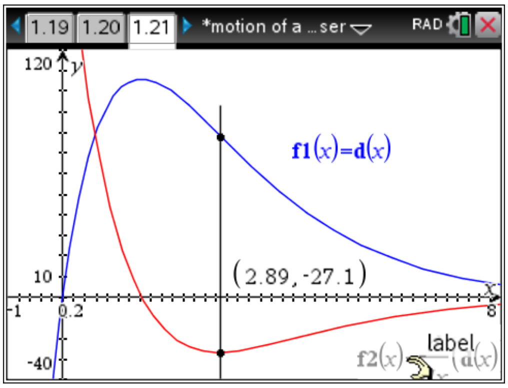


8. In what way do the values of d'(1) and d'(2) correspond to the graph?
   What do the signs of d'(1) and d'(2) tell you about the motion of the door?

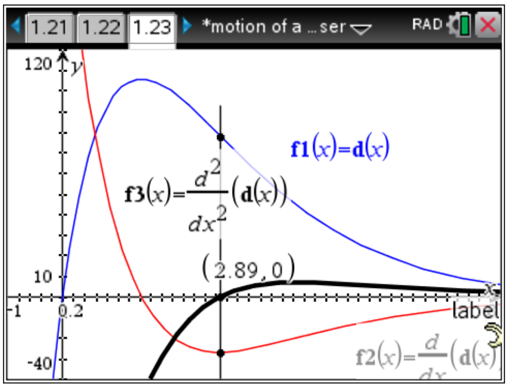
9. Plot the graph of d'(t) on the same set of axes.
   What is true about the graph of d at the point where d'(t)=0? What is happening to the door's motion at this time?

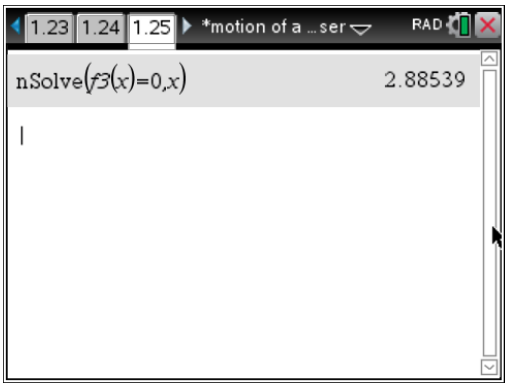


10. Use the minimum feature of your calculator in Analyze graph to find the value of t at which d'(t) is a minimum.
    What does d(t) equal at this value of t?
    Plot a dot at this point on the graph.



11. What do you expect the second derivative d"(2.89) be equal to? Confirm graphically and numerically.





12. What name is given to this point?