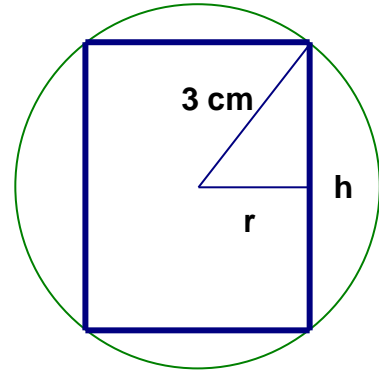


## Modelling with functions CAS Activity.

A cylinder is inscribed in a sphere as shown in the diagram. The radius of a sphere is 3 cm.



- Fully define function  $V_1$ , which gives the volume of a cylinder in terms of its radius  $r$ .
- Fully define function  $V_2$ , which gives the volume of a cylinder in terms of its height  $h$ .
- Sketch both volume functions on GDC.
- State domain and range for each volume function.
- Find the maximum volume of the cylinder.
- Use one of the above functions to find the values of  $x$  for which the volume is greater than  $60 \text{ cm}^3$ .

Using Pythagoras we can form the equation

$$r^2 + \frac{h^2}{4} = 9$$

The volume of a cylinder is given by the formula

$$V = \pi r^2 h$$

a) Expressing  $h$  in terms of  $r$  gives

$$h = \sqrt{36 - 4r^2}$$

$$\therefore V_1 = \pi r^2 \sqrt{36 - 4r^2}$$

$$V_1 : [0, 3] \rightarrow R, V_1(r) = \pi r^2 \sqrt{36 - 4r^2}$$

Class discussion how we restrict domain for  $r$ .

b) Expressing  $r^2$  in terms of  $h$  gives

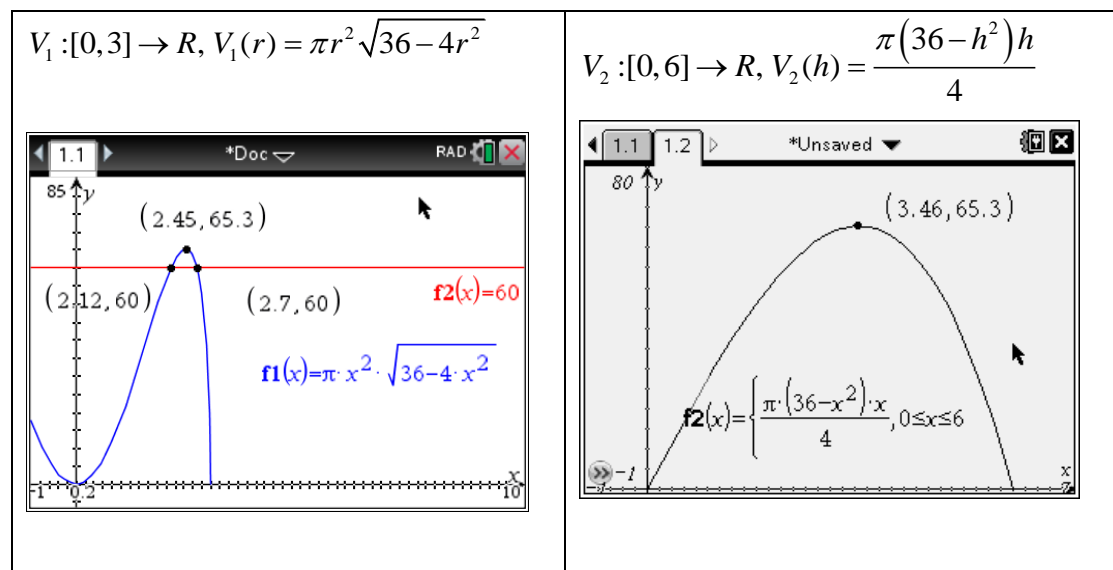
$$r^2 = 9 - \frac{h^2}{4} = \frac{36 - h^2}{4}$$

$$\therefore V_2 = \frac{\pi(36 - h^2)h}{4}$$

$$V_2 : [0, 6] \rightarrow R, V_2(h) = \frac{\pi(36 - h^2)h}{4}$$

Class discussion how we restrict domain for  $h$ .

c)



d)

$V_1(r)$	$V_2(h)$
Domain $0 \leq r \leq 3$	Domain $0 \leq h \leq 6$
Range $0 \leq V_1 \leq 65.3$	Range $0 \leq V_2 \leq 65.3$

e) The maximum volume of the cylinder is  $65.3 \text{ cm}^3$ .

f) Use solve or graphical approach – class discussion.