Applications of differential equations.

Newton's Law of Cooling.

A teacher pours a cup of coffee at lunchtime. The lunchroom is at a constant temperature of 22°C and the coffee is initially 72°C. The coffee becomes undrinkable (too cold) when its temperature drops below 50°C.

After 5 minutes the temperature has fallen to 65°C. Find:

a) How long after it was poured the coffee remains drinkable.
b) The temperature of the coffee after 30 minutes.

Solution:

$$\frac{dT}{dt} = -k(T-22)$$

Use separation of variables method:

$$\frac{dT}{T-22} = -k\,dt$$

$$\ln(T-22) = -k\,t + c$$

$$e^{-kt+c} = T-22$$

$$Ae^{-kt} = T-22$$

$$T = 22 + Ae^{-kt}$$

At $t = 0,\ T = 72 \Rightarrow A = 50$

$$\therefore T = 22 + 50e^{-kt}$$

At $t = 5, T = 65$

$$65 = 22 + 50e^{-5k}$$

$$43 = 50e^{-5k}$$

$$\frac{43}{50} = e^{-5t}$$

$$.86 = e^{-5k}$$

$$\ln(.86) = -5k$$

$$k = -\frac{\ln(0.86)}{5}$$

So $T = 22 + 50e^{-\frac{\ln(0.86)\,t}{5}}$

a)

$$when\ T = 50$$

$$50 = 22 + 50e^{kt}$$

$$\frac{28}{50} = e^{kt}$$

$$\ln\left(\frac{28}{50}\right) = kt$$

$$t = \frac{\ln(.56)}{k}$$

$$t = 19.2$$

b)

when t=30,

$$T = 42.2^{o}C$$

Graphically:

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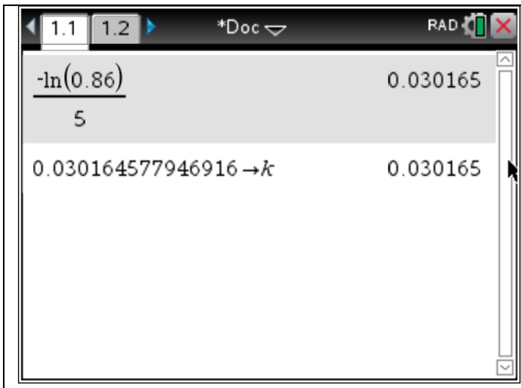
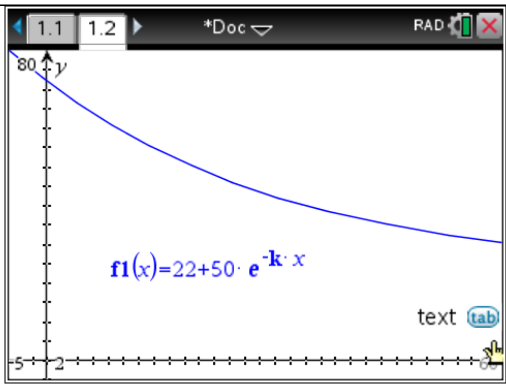
<td>

Store the value of k.</td>
<td>

Sketch the graph.</td>
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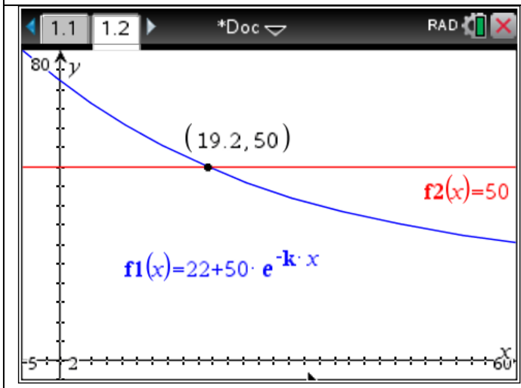
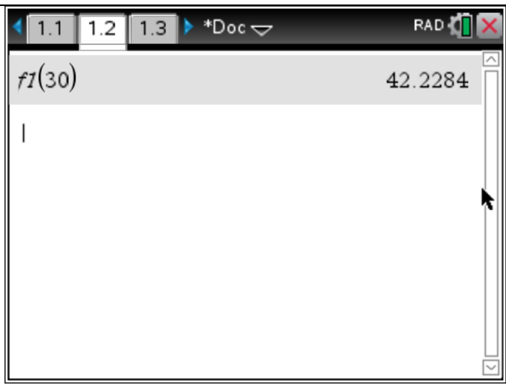
<td>

Coffee becomes undrinkable after 19.2 minutes.</td>
<td>

The temperature of coffee drops to 42.2 degrees after 30 minutes.</td>
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