

INTEGRAL CALCULUS ASSIGNMENT

NAME: _____

Due Date: _____

PART ONE: Technology Free.

Please show *all working* for Technology Free Part.

1. Integrate the following:

a) $\int x^{-5} dx$

b) $\int \sqrt{x} dx$

c)

d) $\int x^{\frac{3}{4}} dx$

$$\int (x^3 + 2x - 6) dx$$

4 marks

2. Evaluate the following definite integrals:

a) $\int_0^1 (x^2 - 3x) dx$

b) $\int_{-1}^1 \sqrt{x} dx$

c) $\int_0^4 x^3 dx$

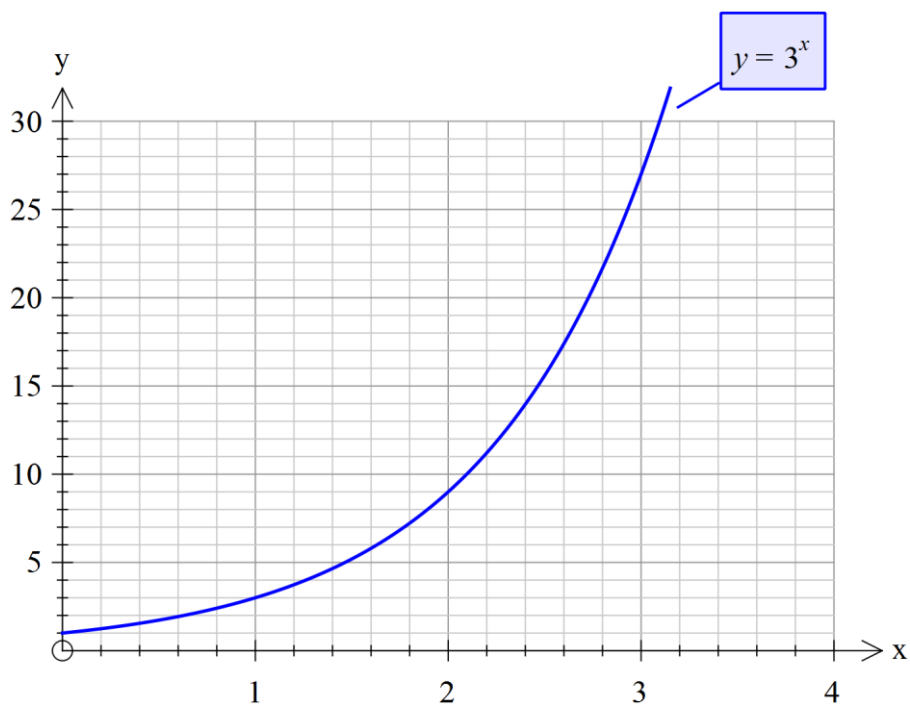
d) $\int_{-2}^1 (x+1)^2 dx$

8 marks

3. Find the exact area bounded by the graph of $y = \sqrt{x}$ between $x = 0$, $x = 2$ and the x -axis.

4 marks

4. Use the left-endpoint estimate to estimate the area under the graph of $y = 3^x$ over the interval $[0, 3]$ with the interval width of 1 unit. Show the rectangles on the graph below.



3 marks

5. Find the positive number p , such that the area of the region bounded by the graph of $f(x) = px(3-x)^2$ and the x -axis is equal to 2 square units.

4 marks

6. A body starts from O and moves in a straight line. After t seconds ($t \geq 0$) its velocity (v cm/s) is given by $v = 3t - 6$.

a) Find its position x in terms of t .

b) Find its position after 3 seconds.

c) What is the distance travelled in the first 3 seconds?

d) Determine its average velocity in the first 3 seconds.

e) Find its average speed in the first 3 seconds.

8 marks

7. Find y in terms of x if:

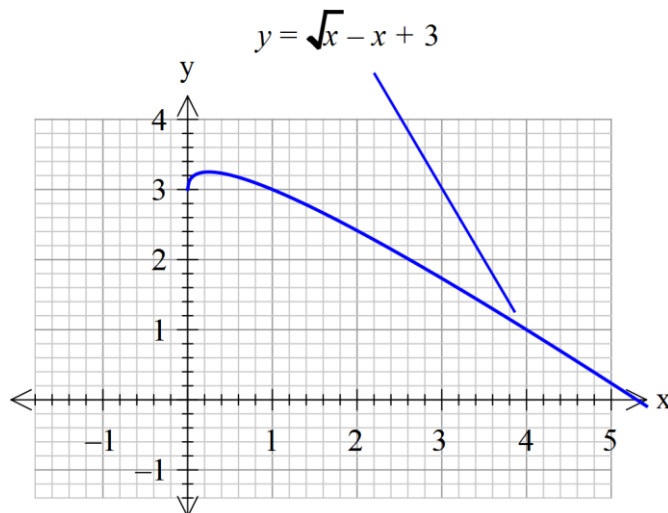
a) $\frac{dy}{dx} = 6$ and $y = 4$ when $x = 1$

b) $\frac{dy}{dx} = 4x^{-1/2}$ and $y = 1$ when $x = 9$

c) $\frac{dy}{dx} = 2\sqrt{x}$ and $y = 1$ when $x = 9$

6 marks

8. Find the exact area under the graph of $y = \sqrt{x} - x + 3$ between $x = 1$ and $x = 4$.

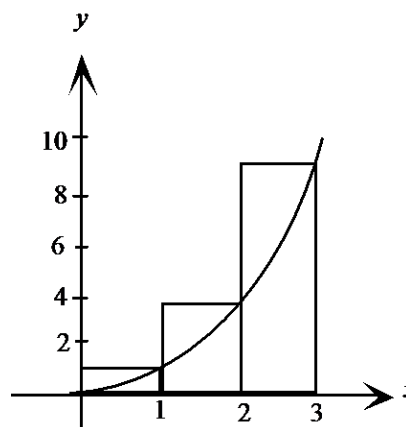


4 marks

PART TWO: Technology Active

8. The graph with equation $y = x^2$ for $x > 0$ is shown. When the area between the graph and the x -axis for $0 \leq x \leq 3$ is approximated by using the upper rectangles and the partitioning as shown, the area is

- A 6
- B 9
- C $9\frac{1}{2}$
- D 10
- E 14



9. If $\int_0^k 4 - 2x + 3x^2 \, dx = 30$, find a value for k .

4 marks

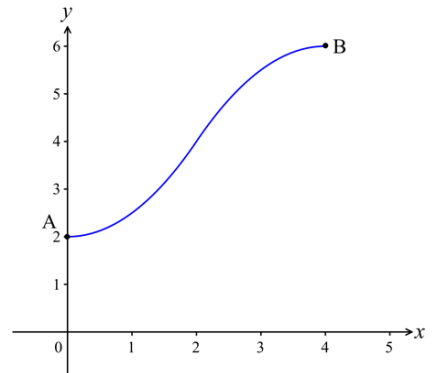
10. $\int_{-1}^1 3f(x) - 1 \, dx$ can be written as

- A $3\int_{-1}^1 f(x) \, dx - 1$
- B $3\int_{-1}^1 f(x) - 1 \, dx$
- C $3\int_{-1}^1 f(x) \, dx - \int_{-1}^1 1 \, dx$
- D $3\int_{-1}^1 f(x) \, dx - 3$
- E $3\int_{-1}^1 f(x) \, dx + 3$

11.

To find the approximate area under the curve shown, Marjorie finds the area of the shape formed by joining $OABC$ using straight lines. The area of the region that Marjorie formed is equal to

- A 8
- B 12
- C 14
- D 16
- E 24



12. A particle moves in a straight line with acceleration of $12t - 5 \text{ m/s}^2$ at time t seconds. The particle has an initial velocity of 1 m/s and an initial position of 0 m from a fixed point O . The velocity of the particle at $t = 1$ is

- A 1 m/s
- B -5 m/s
- C 7 m/s
- D 2 m/s
- E 3 m/s

Extended-response questions

13. The curve with equation $y = f(x)$ has derivative function $f'(x) = 6x^2 + k$, where k is a positive constant.

a) If $\int_0^4 f'(x) dx = m$, find the value of k in terms of m .

b) The point $(1, 10)$ lies on the curve and the tangent at this point passes through the origin.

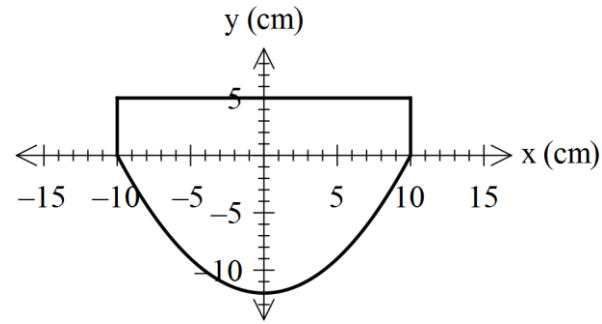
i) Find value of m .

ii) Find the equation of the curve.

14. Concrete is poured from a mixer down a chute which has a cross-sectional shape as shown.

The curved bottom has the shape given by the function

$$f : [-10, 10] \rightarrow \mathbb{R}, \quad f(x) = 0.12x^2 - 12.$$



a) Find the area of the cross-section.

b) What volume of concrete can be poured in one minute? Give your answer in cubic metres, correct to 3 decimal places.

c) How long does it take to empty a mixer holding 12 m^3 of concrete? Give your answer to the nearest second.

9 marks