TOTAL = 45 marks

**Maths Methods Probability Distributions Assignment**

**Name:** SOLNS/MJ

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|---|
| *Give answers as exact values where applicable. Give answers to 3 decimal places unless otherwise indicated.* |

1. A box holds 240 eggs. The probability that an egg is brown is 0.05.

$X \sim B(240, 0.05)$

(a) Find the expected number of brown eggs in the box.

$E(X) = 240 \times 0.05 = 12$ A1

(b) Find the probability that there are 15 brown eggs in the box.

$Pr(X = 15) = 0.073$ A1

(c) Find the probability that there are at least 10 brown eggs in the box.

$Pr(X \geq 10) = 0.764$ A1

2. The following table shows the probability distribution of a discrete random variable $X$.

| $x$ | $-1$ | 0 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | 0.2 | $10k^2$ | 0.4 | $3k$ |

(a) Find the value of $k$.

M1

$0.2 + 10k^2 + 0.4 + 3k = 1$

$k = 0.1$

A1

(b) Find the expected value of $X$.

$E(X) = 1.5$ A1



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3. In a school, $\frac{1}{3}$ of the students travel to school by bus. Five students are chosen at random. Find the probability that exactly 3 of them travel to school by bus.

$X \sim B(5, \frac{1}{3})$

$Pr(X=3) = 0.165$ A1

$\left(\frac{40}{243}\right)$

4. $X$ is a binomial random variable, where the number of trials is 5 and the probability of success of each trial is $p$. Find the values of $p$ if $P(X = 4) = 0.12$.

$X \sim B(5, p)$

$Pr(X=4) = {}^5C_4\, p^4 (1-p)$

${}^5C_4\, p^4 (1-p) = 0.12$ M1

$p = 0.559$ A1 or $p = 0.973$ A1


5. A biased coin is weighted such that the probability of obtaining a head is $\frac{4}{7}$. The coin is tossed 6 times and $X$ denotes the number of heads observed. Find the value of the ratio $\frac{P(X = 3)}{P(X = 2)}$.

$X \sim B(6, \frac{4}{7})$

$\frac{P(X=3)}{P(X=2)} = 1.778$ A1

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6. A discrete random variable $X$ has a probability distribution given in the following table.

| $x$ | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 |
|---|---|---|---|---|---|---|
| $P(X=x)$ | 0.15 | 0.21 | $p$ | $q$ | 0.13 | 0.07 |

(a) If $E(X) = 2.61$, determine the value of $p$ and of $q$, to 2 decimal places.

M1 $0.15 + 0.21 + p + q + 0.13 + 0.07 = 1$

$0.5 \times 0.15 + 1.5 \times 0.21 + 2.5p + 3.5q + 4.5 \times .13 + 5.5 \times 0.07 = 2.61$

Solve on CAS: $p = 0.29$ A1 $q = 0.15$ A1

(b) Calculate VAR $(X)$ to 2 decimal places.

$VAR(X) = 1.44941...^2$

$= 2.10$ A1

7. A company manufactures television sets. They claim that the lifetime of a set is normally distributed with a mean of 80 months and standard deviation of 8 months.

(a) What percentage of television sets break down in less than 72 months? (to the nearest percent).

m1

$Pr(X < 72) = 0.158655...$ $X \sim N(80, 8^2)$

15.87% 16% A1

(b) Calculate the percentage of sets which have a lifetime between 72 months and 90 months. ? (to the nearest percent).

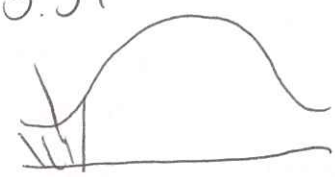
$Pr(72 < X < 90) = 0.735695...$ m1

74% A1

(c) If a set breaks down in less than $x$ months, the company replace it free of charge. They replace 4% of the sets. Find the value of $x$, to the nearest month.

invNorm (0.04, 80, 8) m1

0.04



$x = 66$ A1



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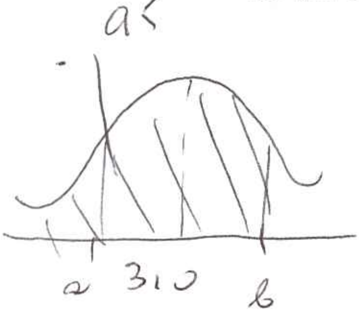
8. It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more.

$X \sim N(310, 30^2)$

$Pr(X > 350) = 0.091$ A1

(b) The probability that the mass of a lion lies between $a$ and $b$ is 0.90, where $a$ and $b$ are symmetric about the mean. Find the value of $a$ and of $b$. Give your answers to the nearest kg.




invNorm$(0.95, 310, 30)$ M1

$b = 359$ A1

$a = 261$ A1

9. The times taken for male runners to complete a marathon can be modelled by a normal distribution with a mean 196 minutes and a standard deviation 24 minutes.

(a) Find the probability that a runner selected at random will complete the marathon in less than 3 hours.

$X \sim N(196, 24^2)$

$Pr(X < 180) = 0.252$ A1

M1

It is found that 5% of the male runners complete the marathon in less than $T_1$ minutes.

(b) Calculate $T_1$, to the nearest minute.

$T_1 = 157$ A1

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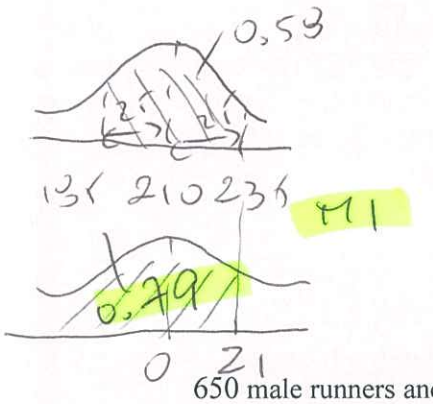
The times taken for female runners to complete the marathon can be modelled by a normal distribution with a mean 210 minutes. It is found that 58% of female runners complete the marathon between 185 and 235 minutes.

(c) Find the standard deviation of the times taken by female runners.




$\sigma = 31.00$ A1

650 male runners and 340 female runners took part in the marathon. 30% of male runners wore sunglasses and 40% of female runners wore sunglasses.

(d) Find the probability that a runner selected at random wears sunglasses.

| | M | M' | |
|---|---|---|---|
| G | 195 | 136 | 331 |
| G' | 455 | 204 | 659 |
| | 650 | 340 | 990 |

M1

$$Pr(G) = \frac{331}{990}$$

$= 0.334$ A1

One runner was selected at random for an interview and was found to wear sunglasses.

(e) Find the probability that the selected runner was male.

$$Pr(M \mid G) = \frac{Pr(M \cap G)}{Pr(G)}$$ M1

$$= \frac{195}{331}$$

$= 0.589$ A1



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10. Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let $X$ denote the number of red balls chosen. The following table shows the probability distribution for $X$.

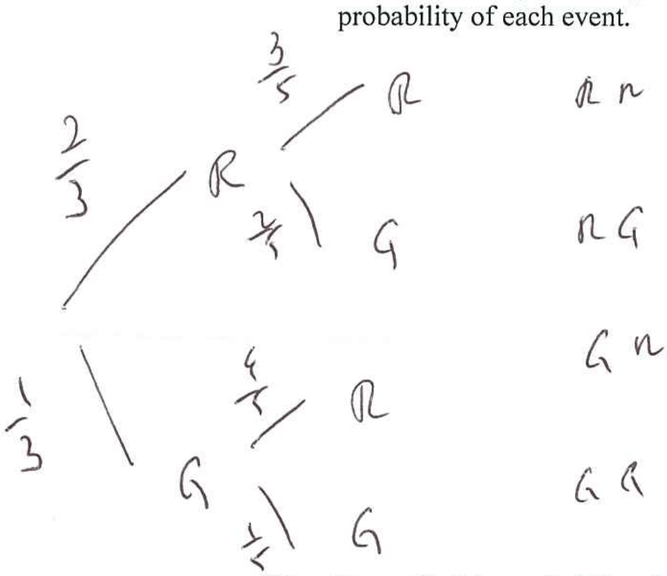
| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X = x)$ | $\frac{3}{10}$ | $\frac{6}{10}$ | $\frac{1}{10}$ |

(a) Calculate E($X$), the mean number of red balls chosen.

$E(x) = \frac{4}{5}$ (0.8) A1

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B, without replacement.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.




M1 A1

(ii) Hence find the probability distribution for $Y$, where $Y$ is the number of red balls chosen. Give your answers as fraction.

| $Y$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(Y = y)$ | $\frac{1}{15}$ | $\frac{8}{15}$ | $\frac{2}{3}$ |

A1



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A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

$\frac{1}{3}$ A $\frac{1}{10}$ RR

$\frac{2}{3}$ B $\frac{2}{5}$ RR

$$\frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{2}{5} = \frac{3}{10}$$

A1

M1

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die.

$$Pr(1 \text{ or } 6 / 2R) = \frac{Pr(1 \text{ or } 6 \cap RR)}{Pr(2R)} = \frac{\frac{1}{3} \times \frac{1}{10}}{\frac{3}{10}} = \frac{1}{9}$$

M1 A1

11. The fish in a lake have weights that are normally distributed with a mean of 1.3 kg and a standard deviation of 0.2 kg.

(a) Determine the probability that a fish that is caught weighs less than 1.4 kg.

$Pr(X < 1.4) = 0.691$ $X \sim N(1.3, 0.2^2)$

A1

(b) John catches 6 fish. Calculate the probability that at least 4 of the fish weigh more than 1.4 kg.

$X \sim B(6, p)$

M1

$p = 1 - 0.69146...$

$= 0.308538$

$Pr(X \geq 4) = 0.077$ A1

(c) Determine the probability that a fish that is caught weighs less than 1 kg, given that it weighs less than 1.4 kg.

$Pr(X < 1 / X < 1.4) = 0.097$ A1

$= \frac{0.066807}{0.691} = 0.0966$

$= 0.097$

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