Name: SOLNS

MATHEMATICAL MODELS TEST

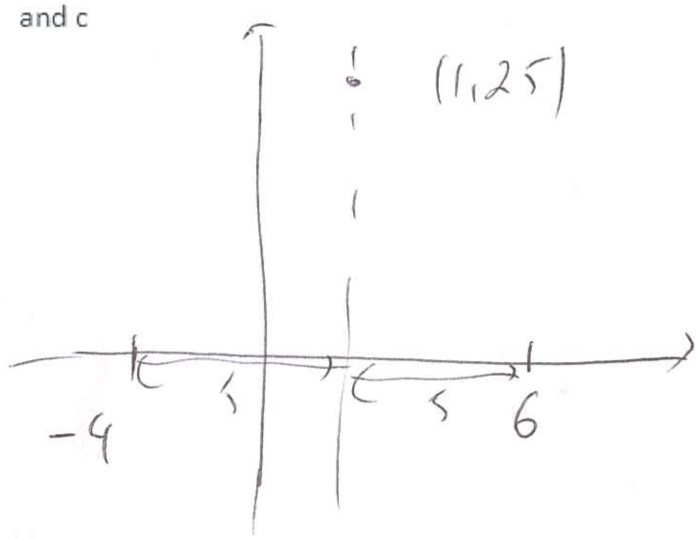
PAPER 1

Question 1

/58

The quadratic function passes through the points (6, 0) and (p, 0)
The maximum point has coordinates (1, 25)

(a) Calculate the value of p [2]
(b) Given that the quadratic function has an equation $y = -x^2 + bx + c$, find b and c [4]



$\boxed{p = -4}$ (M1) A1

b) $-\frac{b}{2a} = 1$ (M1

$-\frac{b}{2(-1)} = 1$

$\boxed{b = 2}$ A1

$y = -(x+4)(x-6)$ M1

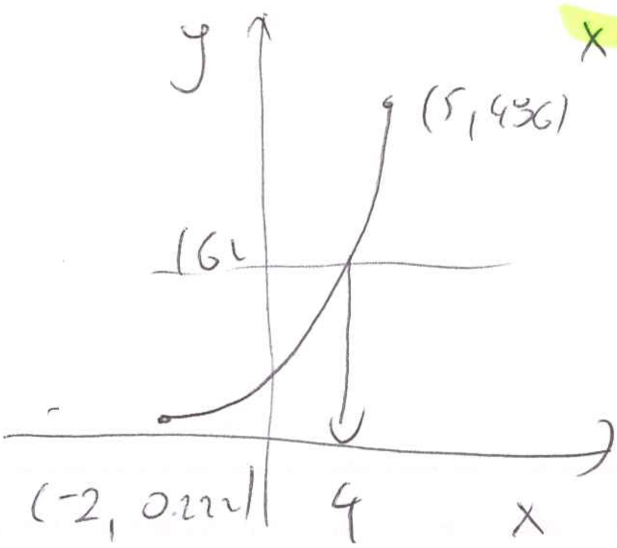
$\boxed{c = 24}$ A1

Question 2

Given the function $f(x) = 2 \times 3^x$ for $-2 \le x \le 5$

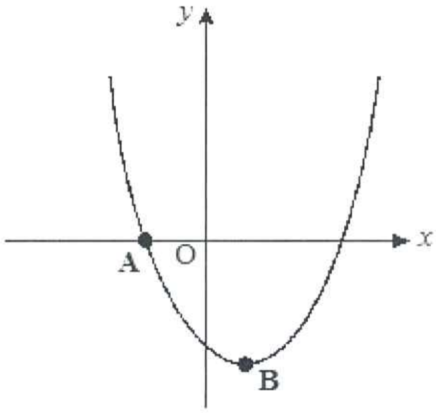
a) Find f(-2) $= 0.222$ A1 [1]
b) Find f(5) $= 486$ A1 [1]
c) Find the range of f(x) $0.222 \le y \le 486$ A1A1 [2]
d) Find the value of x given that f(x) = 162 [2]

$x = 4$ (M1) A1



Question 3

The diagram shows the graph of $y = x^2 - 2x - 3$. The graph crosses the x-axis at the point A, and has a vertex at B.



a) Find the x-intercepts. A1 A1

(-1, 0) (3, 0)

[2]

b) Find the y-intercept.

(0, -3) A1

[1]

c) Find the coordinates of B. (1, -4)

[1]

d) Solve $x^2 - 2x - 3 = 5 - 2x$

$x = 2.83$ and $x = -2.83$ [2]

A1 A1

Question 4

A bacteria's growth rate is given by the equation C = p × $2^{0.5t}$ + q, where t is the time in hours measured from the start of the experiment and p and q are constants.
The biologist places 47 cells in the growth medium at the start of the experiment.
The number of cells in the culture after 4 hours is 53

Use the above information to

a) Write down two equations in p and q [2]
b) Calculate the value of p and of q [2]
c) Find the number of cells in the culture after 10 hours [2]

a) $47 = p + q$ (1) A1

$53 = p \times 2^2 + q$ (2) A1

b) A1 $p = 2$, $q = 45$ A1

c) $C = 2 \times 2^{0.5 \times 10} + 45$ M1

$C(10) = 109$ A1

Question 5

Consider the function $f(x) = 3 + \dfrac{3}{x-1}$.

a) Write the x-intercept.

(0, 0) A1 [1]

b) Write the equation of the horizontal asymptote.

A1 $y = 3$ [1]

c) Write the equation of the vertical asymptote.

A1 $x = 1$ [1]

d) Find

i) $f(5) = 3.75$ A1

ii) $f(-3) = 2.25$ A1

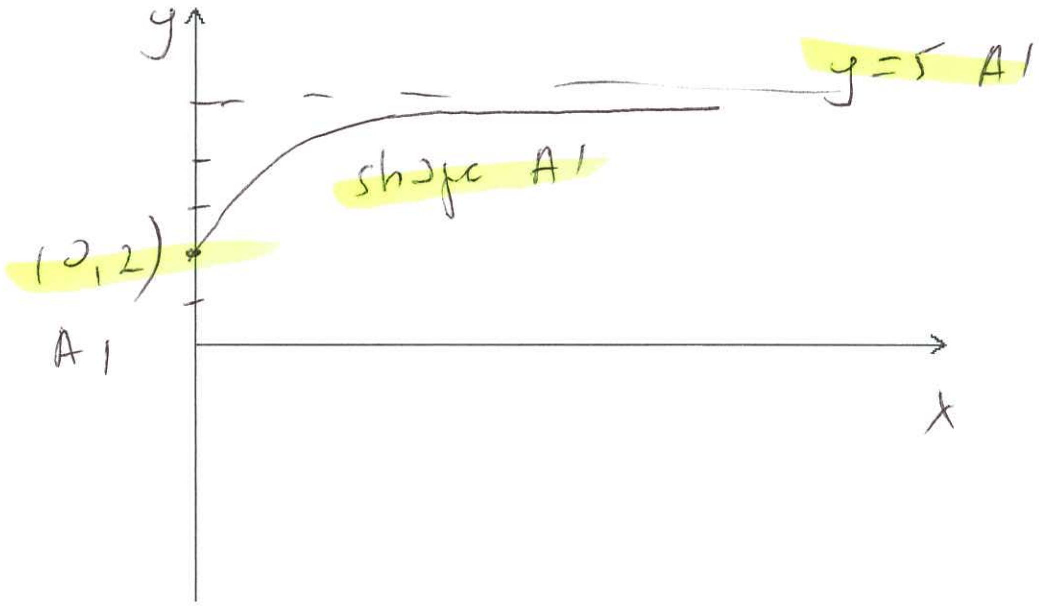
[2]

Question 6

The function $f(x) = 5 - 3(2^{-x})$ is defined for $x \geq 0$.

(a) (i) On the axes below sketch the graph of $f(x)$ and show the behaviour of the curve as x increases.

(ii) Write down the coordinates of any intercepts with the axes.



[4]

(b) Draw the line $y = 5$ on your sketch. A1 [1]

(c) Write down the number of solutions to the equation $f(x) = 5$. [1]

no solutions A1

(asymptote)

PAPER 2

Question 7 [Maximum mark = 11]

Consider the function $f(x) = x^3 + \dfrac{48}{x}$, $x \neq 0$.

(a) Calculate $f(2)$. $= 32$ A1 [1]

(b) Sketch the graph of the function $y = f(x)$ for $-5 \leq x \leq 5$ and $-200 \leq y \leq 200$. [3]

(c) Write down the coordinates of the local minimum. [2]

$(2, 32)$ A1 A1

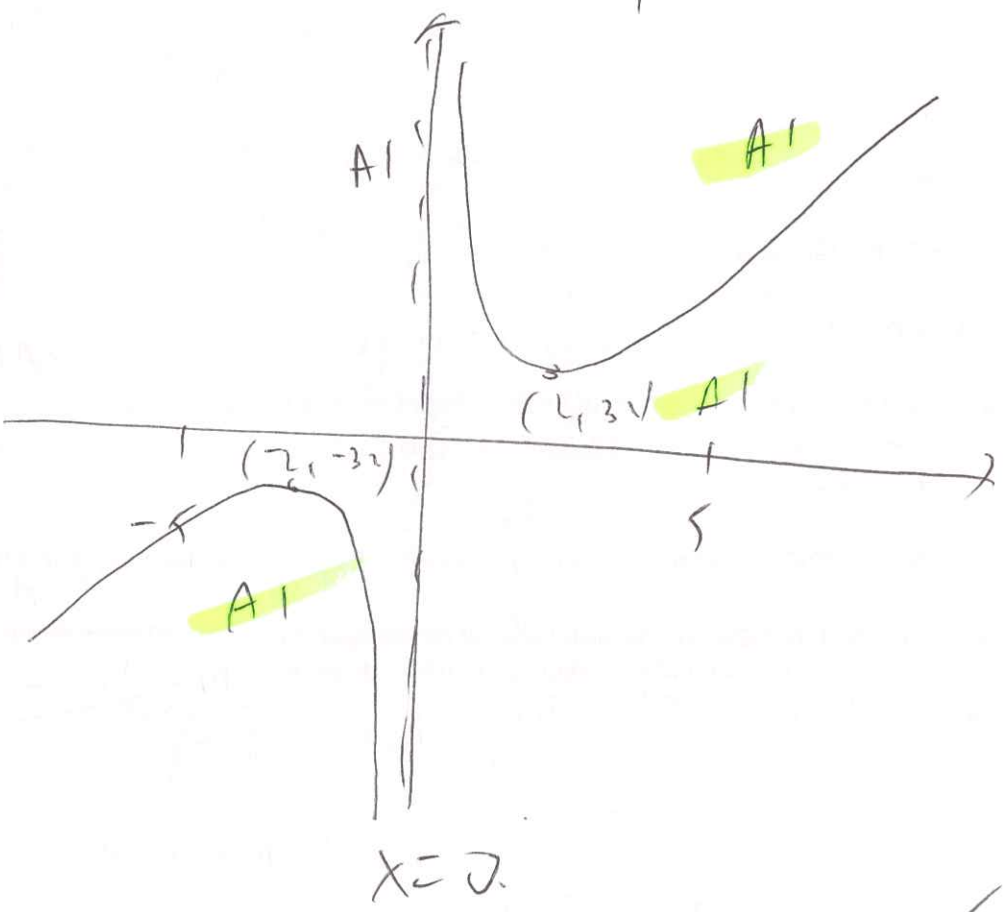
(d) Write down the coordinates of the local maximum. [2]

$(-2, -32)$ A1 A1

(f) Find the range of f. [3]

$y \in \left]-\infty, -32\right] \cup \left[32, \infty\right[$

A1 M1 A1



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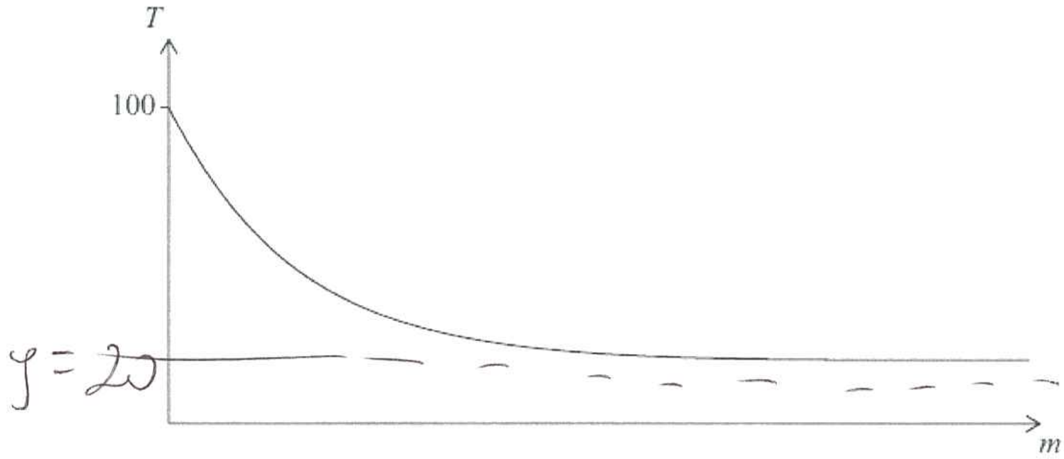
PTO

Question 8

[Maximum mark: 11]

A cup of boiling water is placed in a room to cool. The temperature of the room is $20\,^{\circ}\mathrm{C}$. This situation can be modelled by the exponential function $T = a + b(k^{-m})$, where T is the temperature of the water, in $^{\circ}\mathrm{C}$, and m is the number of minutes for which the cup has been placed in the room. A sketch of the situation is given as follows.



A1

(a) Explain why $a = 20$. room temp, water cannot cool below it. [2]

Initially, at $m = 0$, the temperature of the water is $100\,^{\circ}\mathrm{C}$.

(b) Find the value of b. $100 = \overset{\text{M1}}{20} + b\cdot(k^{0})$ $\quad b = 80$ A1 [2]

After being placed in the room for one minute, the temperature of the water is $84\,^{\circ}\mathrm{C}$.

(c) Show that $k = 1.25$. [2]

M1

(d) Find the temperature of the water three minutes after it has been placed in the room. [2]

$T = 20 + 80\,(1.25)^{-3} = 61.0$ (60.96) A1

(e) Find the total time needed for the water to reach a temperature of $35\,^{\circ}\mathrm{C}$. Give your answer in minutes and seconds, correct to the nearest second.

$20 + 80 \times (1.25)^{-m} = 35$

c) $T = 20 + 80\,k^{-m}$

M1 $84 = 20 + 80\,k^{-1}$

$64 = 80\,k^{-1}$

A1 $\frac{4}{5} = \frac{1}{k}$

$k = \frac{5}{4}$

$k = 1.25$ AG

e) 7 min 30 seconds