# VCE Mathematical Methods – Unit 4 – 2017

Probability SAC : Part A
Technology Active

Student Name

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**Writing Time:** 75 minutes

**QUESTION AND ANSWER BOOKLET**

**Structure of book**

| Number of Tasks | Total Marks |
|---|---|
| **Short Answer: 4** | **36** |

**Time Allocated: 75 minutes writing**

**Materials**

Question and answer booklet. Pens, pencils, erasers.
Students are permitted to bring one bound reference and calculators.

All written responses should be in English.
All answers to be exact values, unless otherwise stated.
Appropriate working must be shown to achieve full marks on questions worth more than one mark.

**Question 1**

Michael Jordan, Lebron James and Kobe Bryant are three professional basketball players playing in 3 point shooting competition. Each player has a different chance of scoring a 3 point shot. The order in which the competition will take place will be Michael to shoot first, Lebron second and Kobe third. The probability of scoring a 3 point shot from Michael is 0.6, whilst it is 0.2 from Lebron and 0.7 from Kobe.

a) Use a tree diagram to illustrate this situation then calculate the probabilities for the sample space.

(2 marks)

$X$ is defined as a discrete random variable denoting the number of **missed** shots.

b) Complete the probability distribution table below.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\Pr(X = x)$ | | | | |

(2 marks)

c) Find Pr $(X \geq 1 \mid X < 3)$ , giving your answer correct to 4 decimal places.

(2 marks)

d) Show that $\mu = E(X) = 1.5$

(1 mark)

e) Find $E(5X + 2)$

(1 mark)

f) Calculate $VAR(X)$.

(2 marks)

g) Hence find the Pr ($\mu - \sigma \leq X \leq \mu + \sigma$) using $\sigma$ correct to two decimal places. Give your answer to two decimal places.

(2 marks)

**Question 2**

Two of the players also compete in a half-court shot competition.
Let $A$ = event that Michael Jordan scores the shot and $B$ = event that Lebron James scores the shot.
If the two events are such that Pr ($A$) = $\frac{1}{5}$, Pr ($B$) = $\frac{1}{3}$ and Pr ($A \cap B$) = $\frac{1}{8}$

a) Use this information to complete the table below. (Give answers in rational form).

| | **A** | **A'** | |
|---|---|---|---|
| **B** | | | |
| **B'** | | | |
| | | | |

(1 mark)

b) Hence find Pr ($A' \cap B$)

(1 mark)

c) Are the events, *A′* and *B,* independent? Justify your answer

(1 mark)

Assume now that A and B are **mutually exclusive** events and that their probabilities remain the same. (Pr (*A*) = $\frac{1}{5}$, Pr (*B*) = $\frac{1}{3}$ )

d) Calculate Pr (*A′* ∩ *B′*).

(2 marks)

**Question 3**

The National Basketball Association (NBA) decided to put on a charity championship match so that the NBA players could show off all their skills. Each of the two team was filled with skilful players eager to win the championship. Throughout the game a number of fouls were committed by players on each team.
The number, *X*, of fouls a player would commit within the game can be modelled by a discrete random variable with probability distribution as shown in the following table:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $\Pr(X = x)$ | 0.3 | $k$ | 0.21 | 0.05 | 0.03 | 0.01 |

a) Find *k*.

(1 mark)

b) If two players are randomly chosen from either team, find the probability that:

**i.** each player has no fouls, correct to 2 decimal places.

(1 mark)

**ii.** the total number of fouls on both players is equal to four, correct to 4 decimal places.

(2 marks)

**iii.** if the total number of fouls on both players is equal to 4, then one of the players has one foul (correct to 4 decimal places).

(2 marks)

c) Eight players are randomly chosen from either team.
Find, correct to four decimal places, the probability that:

**i.** at least three of the players have one foul.

(2 marks)

**ii.** exactly five of the players have one foul, given at least four of the players have one foul.

(2 marks)

d) A random group of players have been selected to receive a participation award for taking part in the charity championship game. Find the smallest number of players in the group if the probability that at least one of the players in the group has less than two fouls is greater than 0.93.

(2 marks)

**Question 4**

There were a population of 600 spectators that turned up to watch the charity championship match. A sample size of 60 spectators were asked if they owned Michael Jordan basketball shoes as he is known to have the best shoes on the market.
$\hat{P}$ is the random variable of the distribution of sample proportion of people who owned his shoes. If the probability of a spectator having his shoes is $p = 0.3$.

a) Find the expected value of $\hat{P}$ .

(1 mark)

b) Find the standard deviation of $\hat{P}$ , correct to 2 decimal places.

(1 mark)

c) Michael Jordan would also like to know how many of the players in the charity championship match will purchase his new shoe which is to be released next month. From a random sample of 20 players, it was found that the sample proportion of people who will buy his shoe is 0.25. Find an approximate 95% confidence interval for the population proportion corresponding to the sample proportion. Give you answer correct to two decimal places.

(2 marks)

d) Michael Jordan is 95% sure that 20% to 30% of spectators at the charity championship match will purchase his new shoe. What sample size was needed for this level of confidence?

(3 marks)

# ADDITIONAL WORKING SPACE