

Logic Exam Questions

Paper 1 Style Questions

1. Consider the following statements:
p: Good mathematics students go to good universities
q: Good music students are good mathematics students
r: Students who go to good universities get good jobs

- (a) From these statements, write two **valid** conclusions.
- (b) Write in words each of the following
- (i) $\neg q$;
(ii) $p \wedge r$.

<p><i>Working:</i></p>	<p><i>Answers:</i></p> <p>(a)</p> <p>.....</p> <p>.....</p> <p>(b) (i)</p> <p>.....</p> <p>(ii)</p> <p>.....</p>
------------------------	--

(Total 4 marks)

2. $[(p \Leftrightarrow q) \wedge p] \Rightarrow q$

- (a) Complete the truth table below for the compound statement above.
(b)

<i>p</i>	<i>q</i>	$p \Leftrightarrow q$	$(p \Leftrightarrow q) \wedge p$	$[(p \Leftrightarrow q) \wedge p] \Rightarrow q$
T	T			
T	F			
F	T			
F	F			

- (c) Explain the significance of your result.
(d)

<p><i>Working:</i></p>	<p><i>Answers:</i></p> <p>(b)</p> <p>.....</p>
------------------------	--

(Total 4 marks)

3. Two propositions p and q are defined as follows:

p : the number ends in zero
 q : the number is divisible by 5

- (a) Write in words
- (i) $p \Rightarrow q$;
 - (ii) the converse of $(p \Rightarrow q)$.
- (b) Write in symbolic form
- (i) the inverse of $(p \Rightarrow q)$;
 - (ii) the contrapositive of $(p \Rightarrow q)$.

<p><i>Working:</i></p>	<p><i>Answers:</i></p> <p>(a) (i)</p> <p>.....</p> <p>(ii)</p> <p>.....</p> <p>(b) (i)</p> <p>(ii)</p>
------------------------	--

(Total 4 marks)

4. Write down the values for **a**, **b**, **c**, **d**, **e** and **f** from the table below:

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \underline{\vee} q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	a			d		
T	F		b				f
F	T			c			
F	F					e	

(Total 6 marks)

5. Three propositions p , q and r are defined as follows:

p : the water is cold. q : the water is boiling. r : the water is warm.

(a) Write one sentence, in words, for the following logic statement:

$$(\neg p \wedge \neg q) \Rightarrow r$$

(b) Write the following sentence as a logic statement using symbols only.

"The water is cold if and only if it is neither boiling nor warm"

<p><i>Working:</i></p>	<p><i>Answers:</i></p> <p>(a)</p> <p>(b)</p>
------------------------	--

Paper 2 Style Questions

6. Let the propositions p , q and r be defined as:

p : Matthew arrives home before six o'clock

q : Matthew cooks dinner

r : Jill washes the dishes

(a) (i) Express the following statement in logical form.
If Matthew arrives home before six o'clock then he will cook dinner.

(1)

(ii) Write the following logic statement in words.

$$\neg q \Rightarrow \neg r$$

(1)

(b) (i) Copy and complete the truth table below.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$	$\neg p$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$
T	T	T						T
T	T	F						T
T	F	T						T
T	F	F						T
F	T	T						T
F	T	F						T
F	F	T						T
F	F	F						T

(5)

(ii) Explain the significance of the truth table above.

(2)

(Total 9 marks)

7. Let $\mathcal{C} = \{x : 1 \leq x < 17, x \in \mathbb{N}\}$.

P , Q and R are the subsets of \mathcal{C} such that

$$P = \{\text{multiples of four}\};$$

$$Q = \{\text{factors of 36}\};$$

$$R = \{\text{square numbers}\}.$$

(a) List the elements of

- (i) \mathcal{C}
- (ii) $P \cap Q \cap R$.

(2)

(b) Describe in words the set $P \cup Q$.

(1)

(c) (i) Draw a Venn diagram to show the relationship between sets P , Q and R .

(2)

(ii) Write the elements of \mathcal{C} in the appropriate places on the Venn diagram.

(3)

(d) Let p , q and r be the statements

$$p: x \text{ is a multiple of four};$$

$$q: x \text{ is a factor of 36};$$

$$r: x \text{ is a square number}.$$

(i) Write a sentence, in words, for the statement

$$(p \vee r) \wedge \neg q$$

(2)

(ii) Shade the region on your Venn diagram in part (c)(i) that represents $(p \vee r) \wedge \neg q$

(1)

(iii) (a) Use a truth table to determine the values of $(p \vee r) \wedge \neg q$. Write the first three columns of your truth table in the following format.

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(3)

(b) Write down one possible value of x for which $(p \vee r) \wedge \neg q$ is true.

(1)

(Total 15 marks)

Logic Exam Questions

1. (a) Award (A1) each for any two of the following:
Good music students go to good universities.
Good mathematics students get good jobs.
Good music students get good jobs. (A2)

(b) (i) There is a good music student who is not a good mathematics student. (A1)

(ii) Good mathematics students go to good universities and students who go to good universities get good jobs. (A1)

OR

Good mathematics students get good jobs. (A1)

[4]

2. (a)

p	q	$p \Leftrightarrow q$	$(p \Leftrightarrow q) \wedge p$	$[(p \Leftrightarrow q) \wedge p] \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	T

(A3)

(b) It is a tautology (or equivalent). The statement is valid. (A1)

[4]

3. (a) (i) *If the number ends in zero then it is divisible by 5* (A1)
(ii) *If the number is divisible by 5 then it ends in zero* (A1)

(b) (i) $\neg p \Rightarrow \neg q$ (A1)

(ii) $\neg q \Rightarrow \neg p$ (A1)

[4]

4. $a = F$ $b = F$ $c = T$ $d = F$ (A1)
 $e = T$ $f = F$ (A1)

[6]

5. (a) *“If the water is not cold and not boiling then it is warm”* (or equivalent statement)(A2)

(b) $p \Leftrightarrow \neg(q \vee r)$ **or** $p \Leftrightarrow \neg q \wedge \neg r$ (A2)

[4]

6. (a) (i) $p \Rightarrow q$ (A1)

(ii) If Matthew doesn't cook dinner then Jill will not wash the dishes (A1) 2

(b) (i) (A5) 5

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$	$\neg p$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$
T	T	T	T	T	F	F	F	T
T	T	F	T	F	T	F	F	T
T	F	T	F	T	F	F	F	T
T	F	F	F	T	T	F	F	T
F	T	T	T	T	F	F	T	T
F	T	F	T	F	T	F	T	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	T	T	T	T

(ii) The truth table is showing that the following argument is valid.

If Matthew arrives home before six o'clock he will cook dinner.

If Matthew cooks dinner then Jill will wash the dishes. Jill did not wash the dishes.

Therefore Matthew did not arrive before six o'clock. (R2) 2

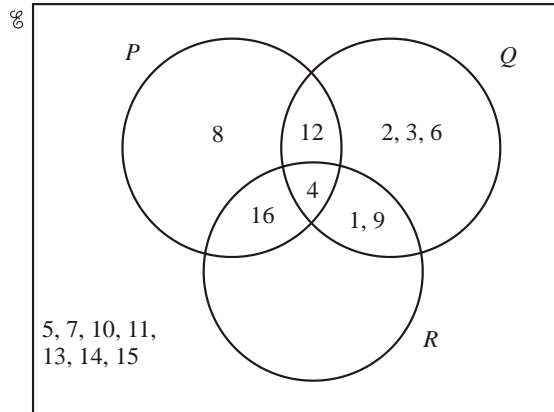
[9]

Mathematics Studies SL

Name _____

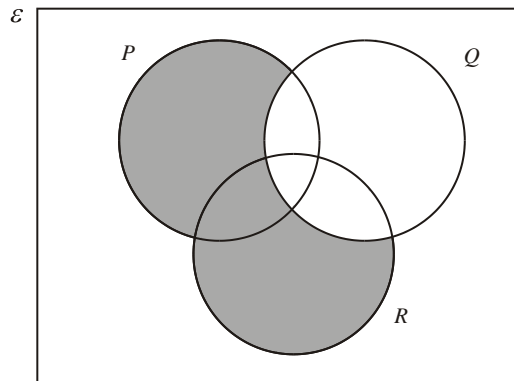
7. (a) (i) $\mathcal{E} = \{1, 2, 3, \dots, 16\}$ (A1)
 (ii) $P \cap Q \cap R = \{4\}$ (A1)
 (b) $P \cup Q$: the set of numbers that are either multiples of 4 or factors of 36, or everything that is in P or Q (or equivalent) (A1)

- (c) (i) (A2)
 (ii) (A3)



- (d) (i) x is a number that is a multiple of 4 or a square number but is not a factor of 36 (A2)

(ii)



(A1)

(iii) (a)

p	q	r	$p \vee r$	$\neg q$	$(p \vee r) \wedge \neg q$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	F	T	F

(A3)

- (b) Either 8 or 16 (A1)

[15]