

# LOGIC

A proposition is a statement that is either true or false but not both.

Proposition		Negation	Compound Propositions				
			And	Or (inclusive)	Or (exclusive)	Implication	Equivalence
$p$	$q$	$\neg p$	$p \wedge q$	$p \vee q$	$p \underline{\vee} q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	F	T	T
T	F	F	F	T	T	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	F	T	T

$p$	$q$	$\neg p$	$\neg q$	Implication $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\neg p \rightarrow \neg q$	Contrapositive $\neg q \rightarrow \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

If two compound propositions have the same truth value then the two propositions are **logically equivalent**. Thus the **implication** and its **contrapositive** are logically equivalent. Also the **converse** and the **inverse** of the implication are logically equivalent.

A **tautology** is a compound proposition that is always true regardless of the truth values of the individual propositions.

A **contradiction** is a compound proposition that is always false regardless of the truth values of the individual propositions.

An **argument** is made up of one or more **premises** that lead to a **conclusion**. The premises and the conclusions are propositions.

In a valid argument the **premises** logically imply the **conclusion** (i.e. the truth values of the logic statement (implication) form a **tautology**).

**Example** Discuss the validity of the argument.

If Sparky is a dog then Sparky is a mammal. Sparky is a dog. Therefore, Sparky is a mammal.

<b>STEP 1:</b> Identify the propositions and name them	$p$ : Sparky is a dog $q$ : Sparky is a mammal
<b>STEP 2:</b> Write the premise and conclusion in symbolic form	<b>Premise:</b> $(p \rightarrow q) \wedge p$ <b>Conclusion:</b> $q$
<b>STEP 3:</b> Draw up a truth table to test validity of argument	<b>Argument:</b> $[(p \rightarrow q) \wedge p] \rightarrow q$ (premise) $\rightarrow$ (conclusion)

$p$	$q$	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$q$	$[(p \rightarrow q) \wedge p] \rightarrow q$
T	T	T	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	F	F	T

Therefore, the argument is valid because the truth values of the logic statement form a tautology.