Specialist Mathematics Unit 3
IMPLICIT DIFFERENTIATION TI *N*-spire CAS.

Explicit Function – is a function in which the dependant variable can be written explicitly in terms of independent variable. For example: $y = \sqrt{4 - x^3}$, $f(x) = \log_e(\sin x)$.

Implicit relation – a function or relation, in which the dependant variable is not isolated on one side of the equation. For example: $x^2 + 3xy - 2y^4 = 1$ represents an implicit relation.

Implicit differentiation is used to differentiate implicit relations.

EXAMPLES:

1. Find $\frac{dy}{dx}$ by implicit differentiation for each of the following relationships:

a. $x = y^3$ b. $x^3 = y^2$ c. $xy = 2x + 1$

d. $x^2 + y^2 = 1$ d. $2x^2 - 2xy + y^2 = 5$

e. $x\,sin^{-1}(y) = e^{2y}$

2. Given that $xy - y - x^2 = 0$, find $\frac{dy}{dx}$

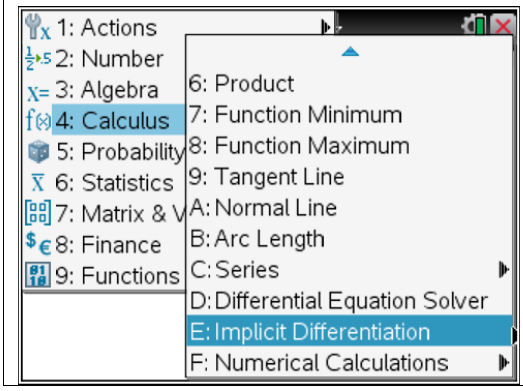
a. by explicit differentiation (making *y* the subject).
b. by implicit differentiation.

3. Find the equation of the tangent to the curve at the indicated point:

a. $y^2 = 8x$ at (2, -4) b. $x^2 - 9y^2 = 9$ at $(5, \frac{4}{3})$

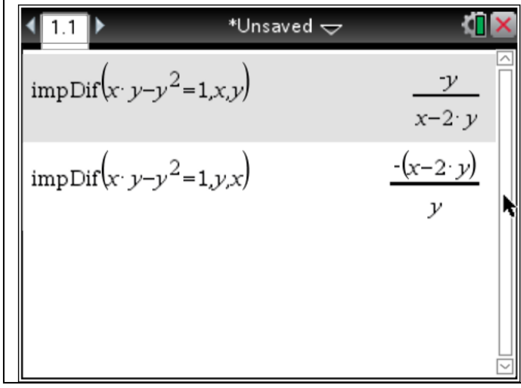
c. $xy - y^2 = 1$ at $(\frac{17}{4}, 4)$ d. $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at (0,-3)

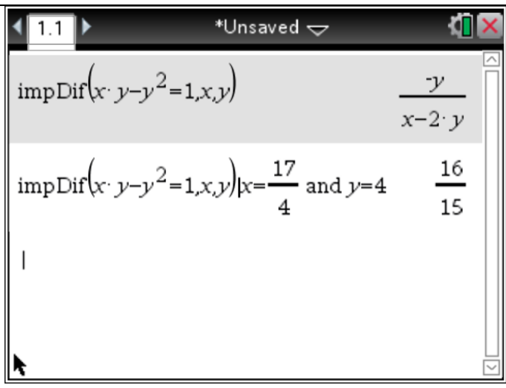
4. Using TI-Nspire CAS calculator:

To differentiate implicitly, in Calculator screen select Menu Calculus Implicit Differentiation : 	Type the equation you want to differentiate: impDif($xy - y \wedge 2 = 1, x, y$) Note that the syntax above gives $\frac{dy}{dx}$.

To find the gradient at a given point type the conditions after using 'given that' sign as shown in the screen to the right.
impDif($xy - y \wedge 2 = 1, x, y) | x = 17/4 \, and \; y = 4$

If you type: impDif($xy - y \wedge 2 = 1, y, x$), you will get $\frac{dx}{dy}$.





Note that you need times sign between *x* and *y*.

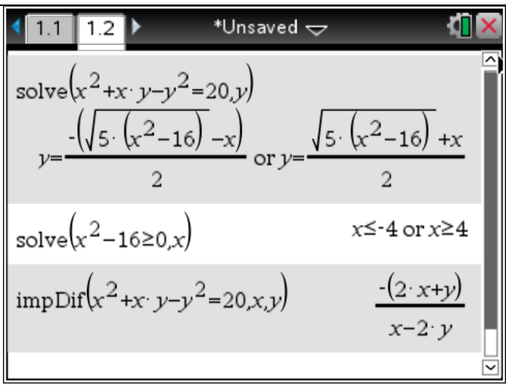
PROBLEM ONE: Consider the conic section with equation $x^2 + xy - y^2 = 20$.

a. Make *y* the subject of the equation.

Note: In the current OS we can draw the conic sections without making *y* the subject.

b. Show that the domain is $(-\infty, -4] \cup [4, \infty)$.

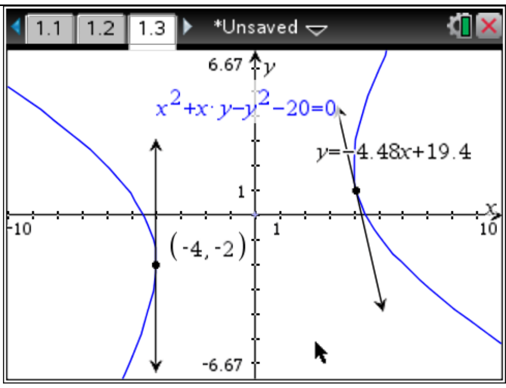
c. Find an expression for $\frac{dy}{dx}$.

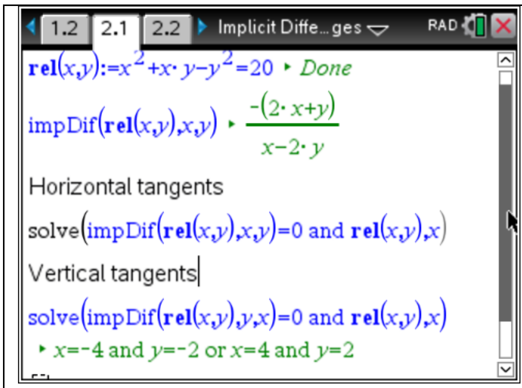


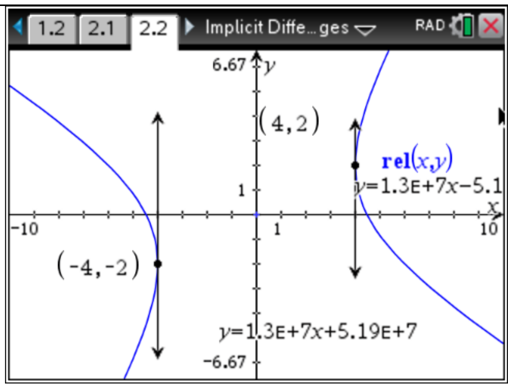
d. Sketch the graph.

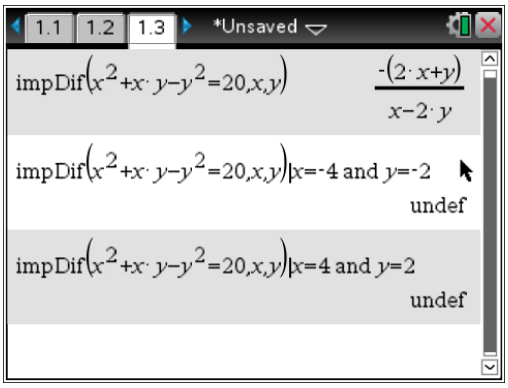
You can also draw equations of tangents to the graph and find their equations.

e. Find the equations of the vertical tangents.









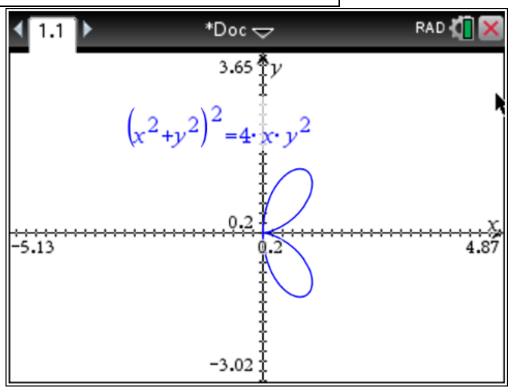
So the equations of the vertical tangents are $x = 4$ *and* $x = -4$.

Horizontal tangents

solve(impDif(rel(x,y),x,y)=0 and rel(x,y),x)

▸ false

No horizontal tangents.



PROBLEM TWO:

The graph of the curve $(x^2 + y^2)^2 = 4xy^2$ is shown alongside.

a. Find the gradient of the curve at the point where $x = 1$. Explain your result.

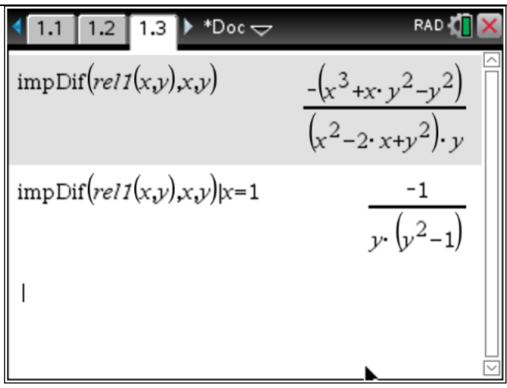
b. Find the gradients of the curve where $y = \frac{1}{2}$, giving your answers to 2 decimal places.

c. Find the equation of the normal

Differentiate implicitly.

Find the *y*-values when $x = 1$ and the *x*-values when $y = \frac{1}{2}$,

It can be seen that $\frac{dy}{dx}$ is undefined for $y = 0$ and also at (1,1) and at (1,-1) – it makes the denominator equal to zero.



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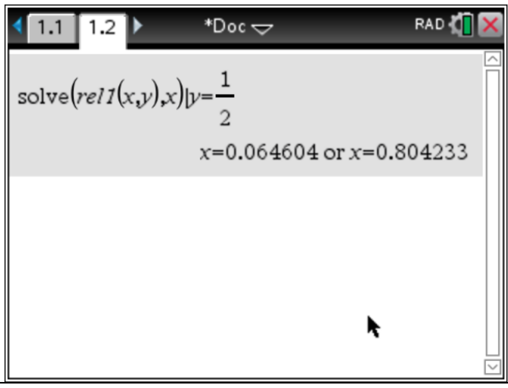
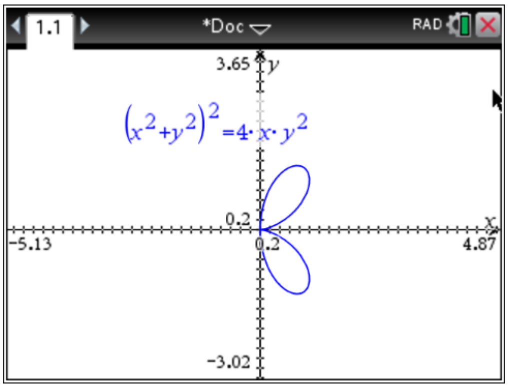
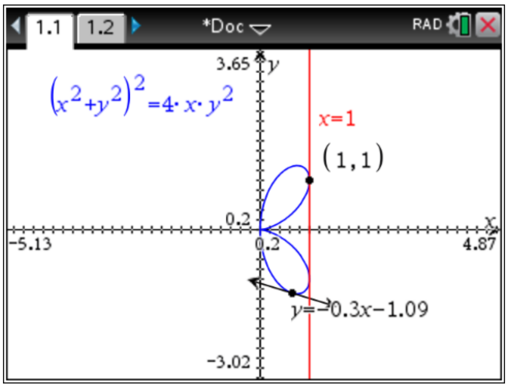
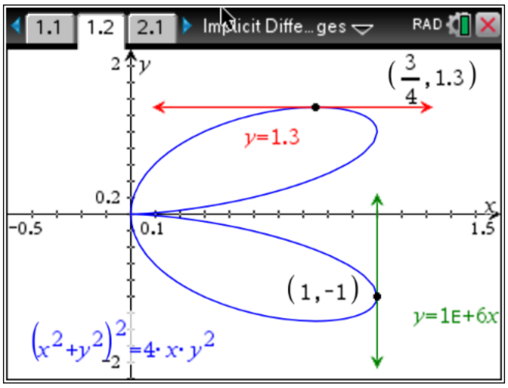
<tr><td></td><td>
</td></tr>
<tr><td>To sketch the graph we need to use Relation entry.</td><td>
</td></tr>
<tr><td>You can also draw tangents to the curve at different values of x.
Vertical tangents occur at x =1,-1

Point on first, then tangent.</td><td>
</td></tr>
</table>

Double Folium with vertical and horizontal tangents.

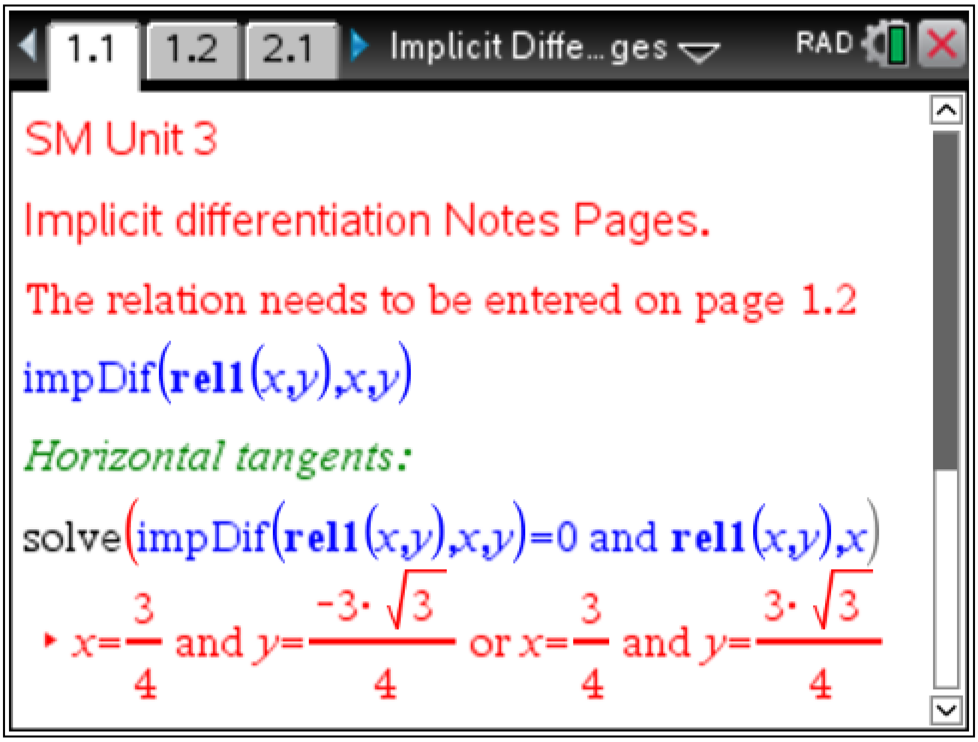


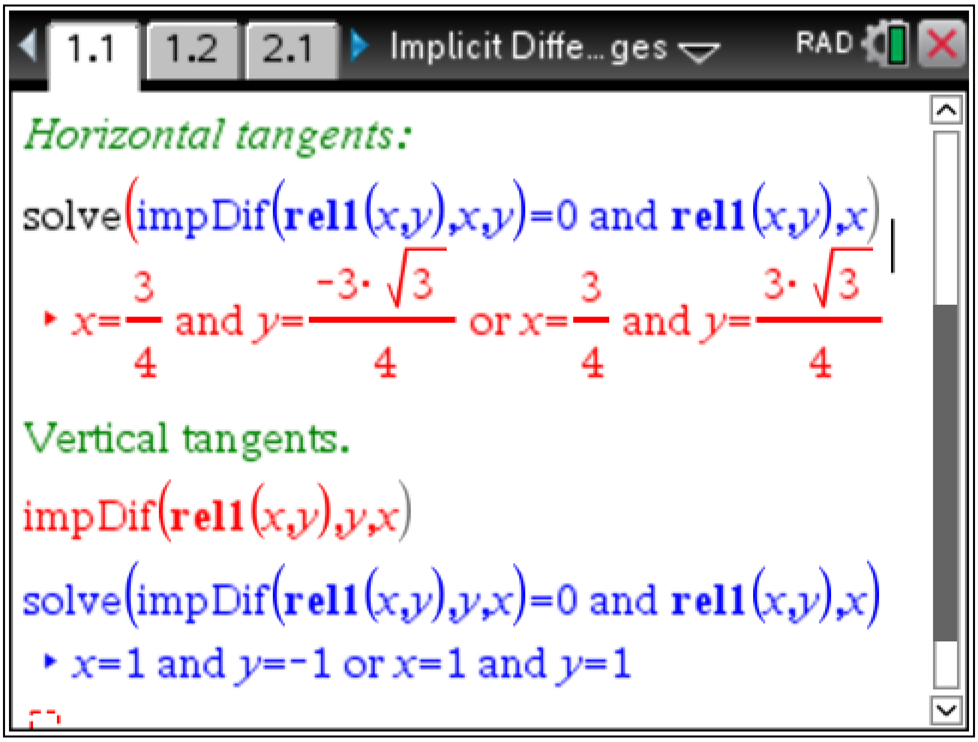
For vertical tangents: $\frac{dx}{dy} = 0$

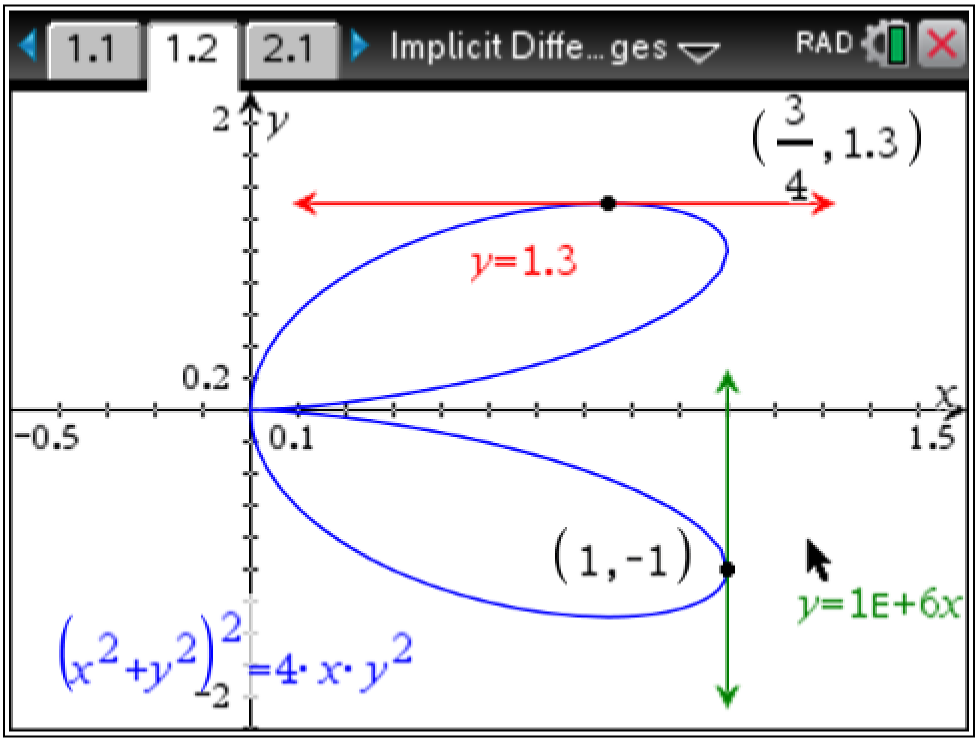
For horizontal tangents: $\frac{dy}{dx} = 0$

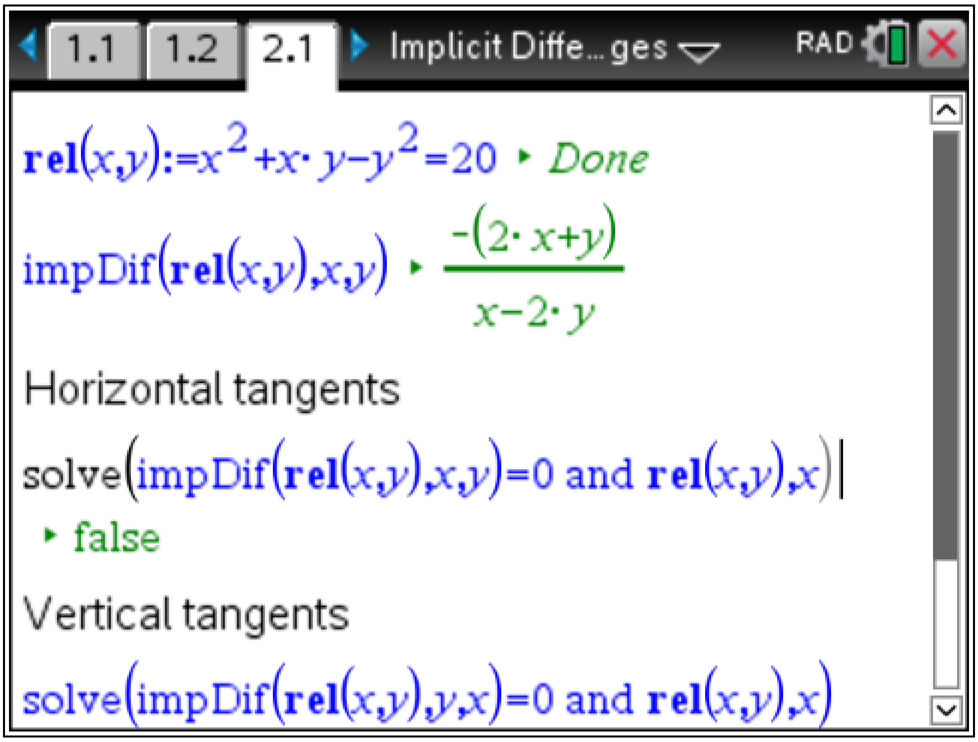
Vertical and horizontal tangents using CAS.

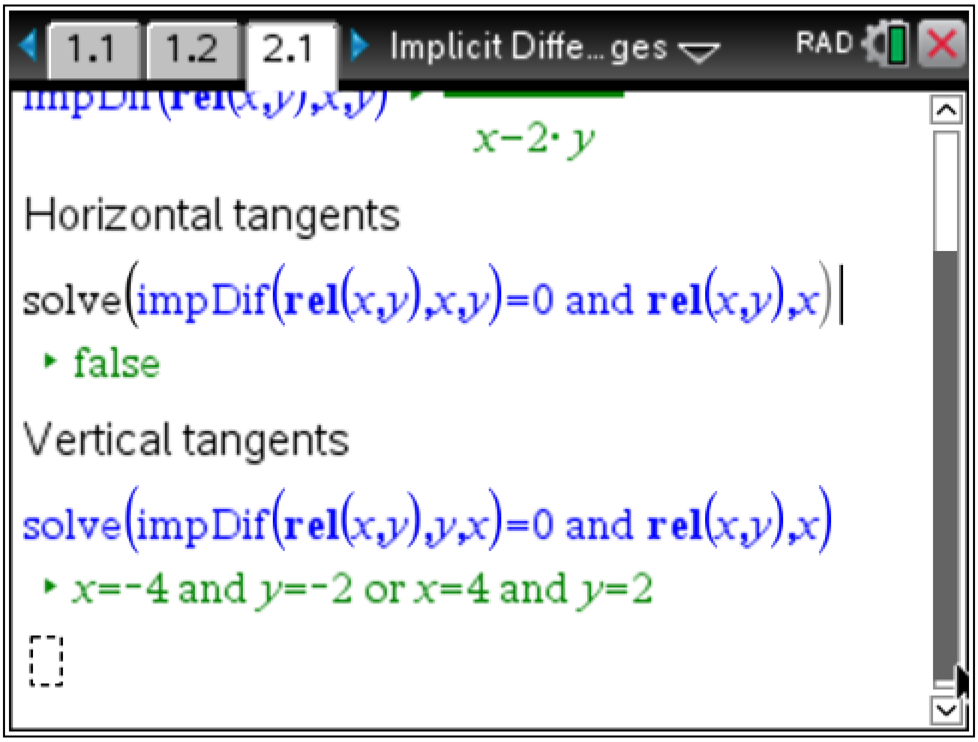
Notes pages for implicit differentiation.

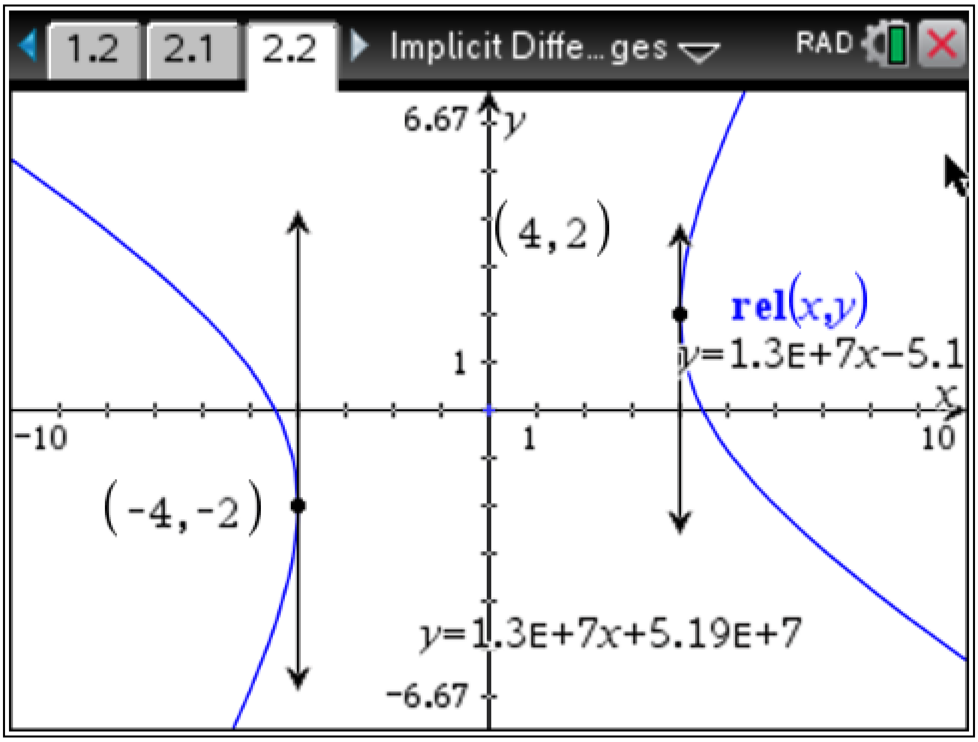



1.1
1.2
2.1
Implicit Diffe...ges
RAD
Horizontal tangents:
solve(impDif(rel1(x,y),x,y)=0 and rel1(x,y),x)
x=3/4 and y=-3·√3/4 or x=3/4 and y=3·√3/4
Vertical tangents.
impDif(rel1(x,y),y,x)
solve(impDif(rel1(x,y),y,x)=0 and rel1(x,y),x)
x=1 and y=-1 or x=1 and y=1


1.1
1.2
2.1
Implicit Diffe...ges
RAD
2
y
(3/4, 1.3)
y=1.3
0.2
x
-0.5
0.1
1.5
(1, -1)
y=1E+6x
(x^2+y^2)^2=4·x·y^2
-2


1.1 1.2 2.1 Implicit Diffe...ges RAD
rel(x,y):=x^2+x·y−y^2=20 ▸ Done
impDif(rel(x,y),x,y) ▸ -(2·x+y)/(x−2·y)
Horizontal tangents
solve(impDif(rel(x,y),x,y)=0 and rel(x,y),x)
▸ false
Vertical tangents
solve(impDif(rel(x,y),y,x)=0 and rel(x,y),x)


1.1 1.2 2.1 Implicit Diffe...ges RAD
x−2·y
Horizontal tangents
solve(impDif(rel(x,y),x,y)=0 and rel(x,y),x)
▸ false
Vertical tangents
solve(impDif(rel(x,y),y,x)=0 and rel(x,y),x)
▸ x=-4 and y=-2 or x=4 and y=2


1.2
2.1
2.2
Implicit Diffe...ges
RAD
6.67
y
(4, 2)
rel(x,y)
y=1.3E+7x−5.1
1
x
−10
1
10
(-4, -2)
y=1.3E+7x+5.19E+7
-6.67