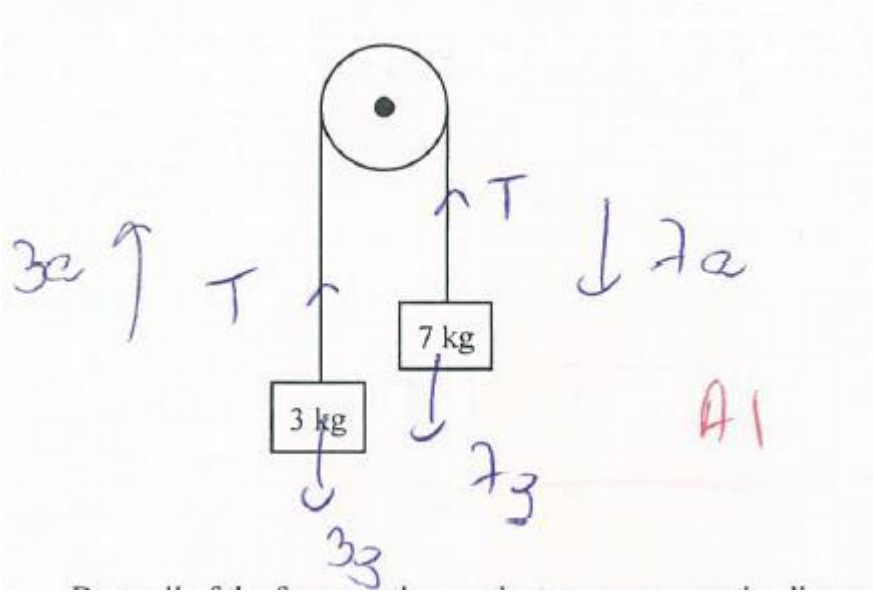
2018 SPECMATH EXAM 1 (NHT) SUGGESTED SOLUTIONS

Question 1

(a)



(b) Write down the equation of motion for each mass:

$7a = 7g - T \quad (1)$

$3a = T - 3g \quad (2)$ M1

Solve

$10a = 4g \Rightarrow a = \dfrac{2g}{5}$

$T = 3 \times \dfrac{2g}{5} + 3g$

$\therefore T = \dfrac{21g}{5}$ A1

Question 2

$|\underset{\sim}{b}| = \sqrt{14}$

$\hat{\underset{\sim}{b}} = \frac{1}{14}\left(2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}\right)$ M1

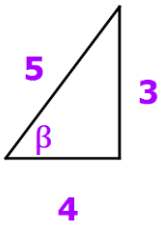
$\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}} = \frac{1}{\sqrt{14}}\left(3\times 2 - 2\times -1 + 3m\right)$

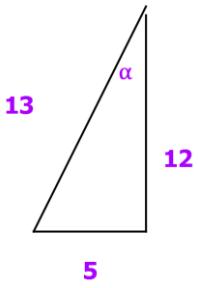
$8 + 3m = \pm 14$

$m = 2 \;\; or \;\; m = -\frac{22}{3}$ A1A1

Question 3

Let $\alpha = \arccos\left(\frac{12}{13}\right)$ and $\beta = \arctan\left(\frac{3}{4}\right)$





$\sin(t) = \sin(\alpha + \beta)$

Use compound angle formula to expand: M1

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(t) = \frac{5}{13}\times\frac{4}{5} + \frac{12}{13}\times\frac{3}{5}$ A1

$\sin(t) = \frac{56}{65}$ A1

Question 4

(a)

$\frac{51.7 - 44.7}{2} = 3.5$

$\text{mean} = 44.7 + 3.5$

$\text{mean} = 48.2$ A1

$\frac{4.2}{3.5} = \sqrt{n}$

$12 = \sqrt{n}$

$n = 144$ A1

(b)

$$\frac{2\times 21}{\sqrt{n}}=1$$

$\sqrt{n}=42$ A1

$n=1764$ A1

Question 5

Complete the square in the denominator

$$x^2+2x+5=(x+1)^2+4$$

Recognise it is arctan and use substitution $u=x+1$

Determine new limits

$x=1,\ u=2$

$x=2\sqrt{3}-1,\ u=2\sqrt{3}$

The integral for *u* becomes:

$$\int_2^{2\sqrt{3}}\frac{1}{u^2+4}du$$ M1

$$\int_2^{2\sqrt{3}}\frac{1}{u^2+4}du=\frac{1}{2}\left(\arctan\left(\frac{u}{2}\right)\right]_2^{2\sqrt{3}}$$ A1

$$=\frac{1}{2}\left(\arctan\left(\frac{2\sqrt{3}}{2}\right)-\arctan(1)\right)$$

$$=\frac{1}{2}\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

$$=\frac{\pi}{24}$$ A1

Question 6

$y=(x-1)e^{2x}$

$\frac{dy}{dx}=e^{2x}(2x-1)$ A1

$\frac{d^2y}{dx^2}=4xe^{2x}$ A1

$a\times 4xe^{2x}+b\times e^{2x}(2x-1)=(x-1)e^{2x}$

$4ax + 2bx - b = x - 1$

$b = 1$ A1

$a = -\frac{1}{4}$ A1

Question 7

(a) $\frac{d}{dx}\left(\left(1-x^2\right)^{\frac{1}{2}}\right) = -\frac{x}{\sqrt{1-x^2}}$ M1A1

(b) $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \sqrt{1+\frac{x^2}{1-x^2}}\, dx$

$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \sqrt{\frac{1-x^2+x^2}{1-x^2}}\, dx$

$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \sqrt{\frac{1}{1-x^2}}\, dx$

$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\, dx$

$= \left[\arcsin(x)\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$ A1

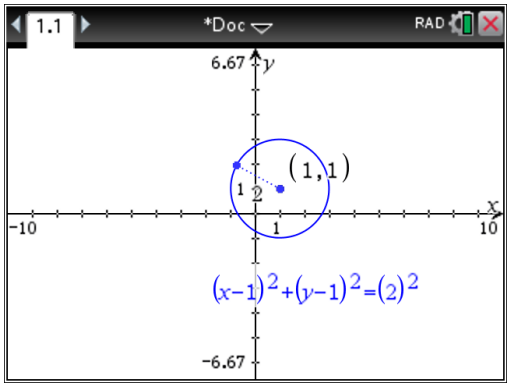
$= \frac{\pi}{3} - \frac{\pi}{6}$

$= \frac{\pi}{6}$ A1

Question 8

(a)

Circle centre (1,1) and radius = 2 **A1**



(b)(i)

$(x-1)^2+(y-1)^2=4$

Differentiate implicitly

$2(x-1)+2(y-1)\dfrac{dy}{dx}=0$ A1

$(y-1)\dfrac{dy}{dx}=-(x-1)$

$\dfrac{dy}{dx}=-\dfrac{x-1}{y-1}$ A1

$=\dfrac{1-x}{y-1}$

(ii)

$x=2,\ 1+(y-1)^2=4$

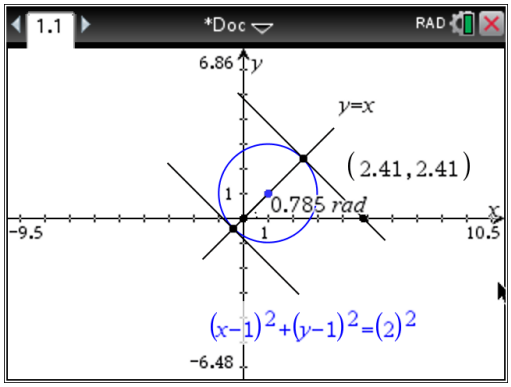
$y-1=\pm\sqrt{3}$

$y=\sqrt{3}+1$ (1st quadrant)

$\dfrac{dy}{dx}=\dfrac{1-2}{\sqrt{3}+1-1}$ A1

$=-\dfrac{1}{\sqrt{3}}$

c) $Arg(z)=\dfrac{\pi}{4}$ *or* $Arg(z)=-\dfrac{3\pi}{4}$ A1A1



Question 9

(a) (i)

$$\frac{1-\tan^2\theta}{2\tan\theta}=a$$

$$1-\tan^2\theta=2a\tan\theta \qquad \text{M1A1}$$

$$1-\tan^2\theta-2a\tan\theta=0$$

$$\tan^2\theta+2a\tan\theta-1=0$$

(ii)

$$\tan\theta=\frac{-2a\pm\sqrt{4a^2+4}}{2}$$

$$=\frac{-2a\pm2\sqrt{a^2+1}}{2} \qquad \text{A1}$$

$$=-a\pm\sqrt{a^2+1}$$

(iii)

$$\tan(2\theta)=\frac{1}{\sqrt{3}}$$

$$2\theta=\frac{\pi}{6}\Rightarrow\theta=\frac{\pi}{12}$$

$$\tan\left(\frac{\pi}{12}\right)=-\sqrt{3}+\sqrt{3+1} \qquad \text{A1}$$

$$=-\sqrt{3}+2$$

b)

$$\frac{dy}{dx}=\sec^2\theta \qquad \text{A1}$$

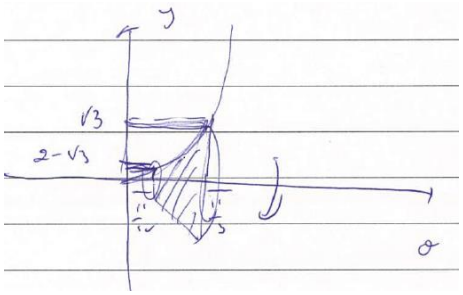
$$\theta=\frac{\pi}{12},\ \frac{dy}{dx}=\sec^2\left(\frac{\pi}{12}\right)$$

$$=1+\tan^2\left(\frac{\pi}{12}\right)$$

$$=1+4-4\sqrt{3}+3$$

$$=8-4\sqrt{3} \qquad \text{A1}$$

c)



$$V = \pi \int_{\frac{\pi}{12}}^{\frac{\pi}{3}} \tan^2\theta \, d\theta \qquad \text{A1}$$

$$= \pi \int_{\frac{\pi}{12}}^{\frac{\pi}{3}} \left(sec^2\theta - 1\right) d\theta$$

$$= \pi \left[\tan\theta - \theta\right]_{\frac{\pi}{12}}^{\frac{\pi}{3}} \qquad \text{A1}$$

$$= \pi \left(\left(\sqrt{3} - \frac{\pi}{3} \right) - 2 + \sqrt{3} + \frac{\pi}{12} \right)$$

$$= \pi \left(2\sqrt{3} - 2 - \frac{\pi}{4} \right) \qquad \text{A1}$$